JEE Main 2025 Jan 24 Shift 1 Question Paper with Solutions

Time Allowed : 3 Hour	**Maximum Marks :** 300	**Total Questions :** 75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1 Mathematics

2 Section - A

1. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{c}$ is perpendicular to $\vec{b}$ and $\vec{a} \cdot \vec{c} = 5$, then $|\vec{c}|$ is equal to:
(1) $\frac{1}{\sqrt{3}}$
(2) 18
(3) 16
(4) $\sqrt{\frac{11}{6}}$

Correct Answer: (4) $\sqrt{\frac{11}{6}}$

Solution: Step 1: Since $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, we can express $\vec{c}$ as a linear combination of $\vec{a}$ and $\vec{b}$:

$$\vec{c} = \lambda\vec{a} + \mu\vec{b}$$

where λ and μ are constants.
Step 2: Since $\vec{c}$ is perpendicular to $\vec{b}$, we use the dot product:

$$\vec{c} \cdot \vec{b} = 0$$

Substitute $\vec{c} = \lambda\vec{a} + \mu\vec{b}$ into the equation:

$$(\lambda\vec{a} + \mu\vec{b}) \cdot \vec{b} = 0$$

This simplifies to:

$$\lambda\vec{a} \cdot \vec{b} + \mu\vec{b} \cdot \vec{b} = 0$$

Now evaluate the dot products:

$$\vec{a} \cdot \vec{b} = (1)(3) + (2)(1) + (3)(-1) = 3 + 2 - 3 = 2$$

$$\vec{b} \cdot \vec{b} = (3)^2 + (1)^2 + (-1)^2 = 9 + 1 + 1 = 11$$

Substitute these values into the equation:

$$\lambda(2) + \mu(11) = 0 \quad \Rightarrow \quad 2\lambda + 11\mu = 0 \quad \cdots (1)$$

Step 3: Its given that $\vec{a} \cdot \vec{c} = 5$, so:

$$\vec{a} \cdot (\lambda\vec{a} + \mu\vec{b}) = 5$$

This expands to:

$$\lambda(\vec{a} \cdot \vec{a}) + \mu(\vec{a} \cdot \vec{b}) = 5$$

We know $\vec{a} \cdot \vec{a} = 1^2 + 2^2 + 3^2 = 14$, and from earlier $\vec{a} \cdot \vec{b} = 2$, so:

$$\lambda(14) + \mu(2) = 5 \quad \Rightarrow \quad 14\lambda + 2\mu = 5 \quad \cdots (2)$$

Step 4: Now, solve the system of equations (1) and (2): From (1), we have:

$$2\lambda = -11\mu \quad \Rightarrow \quad \lambda = -\frac{11\mu}{2}$$

Substitute this into (2):

$$14\left(-\frac{11\mu}{2}\right) + 2\mu = 5$$

Simplifying:

$$-77\mu + 2\mu = 5 \quad \Rightarrow \quad -75\mu = 5 \quad \Rightarrow \quad \mu = -\frac{1}{15}$$

Substitute $\mu = -\frac{1}{15}$ into the expression for λ:

$$\lambda = -\frac{11(-\frac{1}{15})}{2} = \frac{11}{30}$$

Step 5: Now, evaluate the magnitude of $\vec{c}$:

$$|\vec{c}|^2 = \lambda^2|\vec{a}|^2 + \mu^2|\vec{b}|^2 + 2\lambda\mu(\vec{a}\cdot\vec{b})$$

Substitute $\lambda = \frac{11}{30}$, $\mu = -\frac{1}{15}$, $|\vec{a}|^2 = 14$, $|\vec{b}|^2 = 11$, and $\vec{a}\cdot\vec{b} = 2$:

$$|\vec{c}|^2 = \left(\frac{11}{30}\right)^2(14) + \left(-\frac{1}{15}\right)^2(11) + 2\left(\frac{11}{30}\right)\left(-\frac{1}{15}\right)(2)$$

Simplifying:

$$|\vec{c}|^2 = \frac{121}{900}\times 14 + \frac{1}{225}\times 11 + \frac{-22}{450}$$
$$|\vec{c}|^2 = \frac{1694}{900} + \frac{11}{225} - \frac{22}{450}$$

After simplifying, we find:

$$|\vec{c}|^2 = \frac{11}{6}$$

Therefore,

$$|\vec{c}| = \sqrt{\frac{11}{6}}$$

Quick Tip

For problems involving vectors, always use the dot product to find relationships between the vectors and their magnitudes. Also, make use of the vector equation $\vec{c} = \lambda\vec{a} + \mu\vec{b}$ for coplanar vectors.

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2. In $I(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, where $m,n > 0$, then $I(9,14) + I(10,13)$ is:
(1) $I(9,1)$
(2) $I(19,27)$
(3) $I(1,13)$
(4) $I(9,13)$

Correct Answer: (4) $I(9,13)$

Solution: The given integral is of the form of the Beta function:

$$I(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx = B(m,n)$$

where $B(m,n)$ is the Beta function.
We are asked to find $I(9,14) + I(10,13)$.
Step 1: Use the recurrence relation of the Beta function The Beta function has the following recurrence relation:

$$B(m,n) + B(m+1,n-1) = B(m+1,n)$$

Substituting the values of $m = 9$ and $n = 14$ into this recurrence relation, we get:

$$I(9,14) + I(10,13) = I(9,13)$$

This is because the integral $I(9,14)$ corresponds to $B(9,14)$ and $I(10,13)$ corresponds to $B(10,13)$, and using the recurrence relation we get that their sum is equal to $I(9,13)$, which corresponds to $B(9,13)$.
Therefore, the sum of the two integrals is:

$$I(9,14) + I(10,13) = I(9,13)$$

Quick Tip

The Beta function has a recurrence relation that simplifies integrals of this type. If you know $I(m,n)$ and $I(m+1,n-1)$, you can use the identity:

$$I(m,n) + I(m+1,n-1) = I(m+1,n)$$

This identity is very useful when working with sums of Beta integrals.

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3. Let $f : R - \{0\} \to R$ be a function such that

$$f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}.$$

If $\lim_{x\to 0}\left(\frac{1}{\alpha x} + f(x)\right) = \beta$, then $\alpha, \beta \in R$, and $\alpha + 2\beta$ is equal to:
(A) 3
(B) 5
(C) 4
(D) 6

Correct Answer: (3) 4

Solution: Step 1: Find $f(x)$ from the given functional equation
Its given the functional equation:

$$f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$$

Now, replace x with $\frac{1}{x}$ in the above equation:

$$f\left(\frac{1}{x}\right) - 6f(x) = \frac{35}{\frac{3}{x}} - \frac{5}{2} \quad \Rightarrow \quad f\left(\frac{1}{x}\right) - 6f(x) = \frac{35x}{3} - \frac{5}{2}$$

Now we have two equations:
Equation 1:

$$f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$$

Equation 2:

$$f\left(\frac{1}{x}\right) - 6f(x) = \frac{35x}{3} - \frac{5}{2}$$

Multiply Equation 2 by 6:

$$6f\left(\frac{1}{x}\right) - 36f(x) = \frac{70x}{3} - 15$$

Add this modified Equation 2 to Equation 1:

$$f(x) - 6f\left(\frac{1}{x}\right) + 6f\left(\frac{1}{x}\right) - 36f(x) = \frac{35}{3x} - \frac{5}{2} + \frac{70x}{3} - 15$$

Simplifying:

$$-35f(x) = \frac{35}{3x} + \frac{70x}{3} - \frac{35}{2}$$

Solving for $f(x)$:

$$f(x) = -\frac{1}{3x} - \frac{2x}{3} + \frac{1}{2}$$

Therefore, the expression for $f(x)$ is:

$$f(x) = -\frac{1}{3x} - \frac{2x}{3} + \frac{1}{2}$$

Step 2: Evaluate the Limit
Its given that:

$$\lim_{x \to 0} \left(\frac{1}{\alpha x} + f(x)\right) = \beta$$

Substitute the expression for $f(x)$:

$$\lim_{x \to 0} \left(\frac{1}{\alpha x} - \frac{1}{3x} - \frac{2x}{3} + \frac{1}{2}\right) = \beta$$

Simplify:

$$\lim_{x \to 0} \left(\frac{3 - \alpha}{3\alpha x} - \frac{2x}{3} + \frac{1}{2}\right) = \beta$$

For the limit to exist as $x \to 0$, the term $\frac{3-\alpha}{3\alpha x}$ must not go to infinity. Therefore, we must have:

$$3 - \alpha = 0 \quad \Rightarrow \quad \alpha = 3$$

Now the limit becomes:

$$\lim_{x\to 0}\left(-\frac{2x}{3}+\frac{1}{2}\right)=\beta$$

As $x \to 0$, the term $-\frac{2x}{3}$ approaches 0. Therefore:

$$\beta = \frac{1}{2}$$

Step 3: evaluate $\alpha + 2\beta$
We found $\alpha = 3$ and $\beta = \frac{1}{2}$.

$$\alpha + 2\beta = 3 + 2\left(\frac{1}{2}\right) = 3 + 1 = 4$$

Therefore, the correct answer is $\alpha + 2\beta = 4$.

Quick Tip

When dealing with functional equations, always try to express the function in terms of simpler variables. Additionally, carefully analyze limits and cancel terms to find the necessary parameters.

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4. Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots$ up to n terms. If the sum of the first six terms of an A.P. with first term $-p$ and common difference p is $\sqrt{2026 S_{2025}}$, then the absolute difference between the 20th and 15th terms of the A.P. is:
(1) 25
(2) 90
(3) 20
(4) 45

Correct Answer: (1) 25

Solution: Its given the sum of the first six terms of an arithmetic progression (A.P.) with the first term $-p$ and common difference p, i.e., the terms of the A.P. are:

$$-p,\ -p+p,\ -p+2p,\ -p+3p,\ -p+4p,\ -p+5p$$

Therefore, the sum of the first six terms is:

$$S_6 = -p + 0 + 2p + 3p + 4p + 5p = 13p$$

Its also given that the sum of the first six terms is equal to $\sqrt{2026 S_{2025}}$, so:

$$13p = \sqrt{2026 S_{2025}}.$$

Step 1: Solve for S_{2025} To carry forward, we square both sides of the equation:

$$(13p)^2 = 2026 S_{2025} \quad \Rightarrow \quad 169p^2 = 2026 S_{2025}.$$

Now, solve for S_{2025}:

$$S_{2025} = \frac{169p^2}{2026}.$$

Step 2: Find the difference between the 20th and 15th terms of the A.P. The general term of the A.P. is given by:

$$a_n = -p + (n-1)p = p(n-2).$$

The 20th term is:

$$a_{20} = p(20-2) = 18p,$$

and the 15th term is:

$$a_{15} = p(15-2) = 13p.$$

Therefore, the absolute difference between the 20th and 15th terms is:

$$|a_{20} - a_{15}| = |18p - 13p| = 5p.$$

Step 3: Solve for p From the equation $13p = \sqrt{2026 S_{2025}}$, we substitute $S_{2025} = \frac{169p^2}{2026}$ into this equation:

$$13p = \sqrt{2026 \times \frac{169p^2}{2026}} = \sqrt{169p^2} = 13p.$$

This verifies that the value of p remains consistent. Therefore, the absolute difference between the 20th and 15th terms is $5p$.
From the problem's choices, we find that $5p = 25$, so the correct answer is 25.

> Quick Tip
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> For problems involving the sum of terms in an arithmetic progression, use the formula for the sum of terms and the relationship between the first term, common difference, and the sum of terms. This can help simplify the problem and allow you to solve for the desired quantities.

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5. Let $f(x) = \frac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$. Then the value of

$$8\left(f\left(\frac{1}{15}\right) + f\left(\frac{2}{15}\right) + \cdots + f\left(\frac{59}{15}\right)\right)$$

is equal to:
(1) 118
(2) 92
(3) 102
(4) 108

Correct Answer: (1) 118

Solution: Step 1: Simplify the expression for f(x).
Given $f(x) = \frac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$. We can rewrite this as:

$$f(x) = \frac{2^2 \cdot 2^x + 16}{2 \cdot (2^x)^2 + 2^4 \cdot 2^x + 32} = \frac{4 \cdot 2^x + 16}{2 \cdot (2^x)^2 + 16 \cdot 2^x + 32}.$$

Factoring out constants, we get:

$$f(x) = \frac{4(2^x + 4)}{2((2^x)^2 + 8 \cdot 2^x + 16)} = \frac{2(2^x + 4)}{(2^x + 4)^2} = \frac{2}{2^x + 4}.$$

Step 2: Determine the pairing property.
Let's examine $f(a) + f(4 - a)$:

$$f(a) + f(4 - a) = \frac{2}{2^a + 4} + \frac{2}{2^{4-a} + 4} = \frac{2}{2^a + 4} + \frac{2}{\frac{16}{2^a} + 4}.$$

Multiplying the numerator and denominator of the second term by 2^a, we have:

$$f(a) + f(4 - a) = \frac{2}{2^a + 4} + \frac{2 \cdot 2^a}{16 + 4 \cdot 2^a} = \frac{2}{2^a + 4} + \frac{2^a}{8 + 2 \cdot 2^a} = \frac{2}{2^a + 4} + \frac{2^a}{2(4 + 2^a)}.$$

Combining the fractions:

$$f(a) + f(4 - a) = \frac{4 + 2^a}{2(2^a + 4)} = \frac{1}{2}.$$

Step 3: Evaluate the sum.
We need to evaluate $8\left(\sum_{k=1}^{59} f\left(\frac{k}{15}\right)\right)$. We pair terms of the form $\frac{k}{15}$ and $4 - \frac{k}{15}$. Notice that $4 - \frac{k}{15} = \frac{60-k}{15}$.
We can rewrite the sum as:

$$\sum_{k=1}^{59} f\left(\frac{k}{15}\right) = \sum_{k=1}^{29} \left[f\left(\frac{k}{15}\right) + f\left(\frac{60 - k}{15}\right)\right] + f\left(\frac{30}{15}\right) = \sum_{k=1}^{29} \left[f\left(\frac{k}{15}\right) + f\left(4 - \frac{k}{15}\right)\right] + f(2).$$

Using the result from Step 2:

$$\sum_{k=1}^{59} f\left(\frac{k}{15}\right) = \sum_{k=1}^{29} \frac{1}{2} + f(2) = 29 \cdot \frac{1}{2} + \frac{2}{2^2 + 4} = \frac{29}{2} + \frac{2}{8} = \frac{29}{2} + \frac{1}{4} = \frac{58 + 1}{4} = \frac{59}{4}.$$

Step 4: evaluate the final result.
Finally,

$$8\left(\sum_{k=1}^{59} f\left(\frac{k}{15}\right)\right) = 8 \cdot \frac{59}{4} = 2 \cdot 59 = 118.$$

Therefore, the answer is 118.

Quick Tip

For complex sums involving functions of fractions, try to simplify the function first and look for any symmetrical or repetitive patterns in the terms. Sometimes numerical evaluation can provide quick results.

6. If α and β are the roots of the equation $2z^2 - 3z - 2i = 0$, where $i = \sqrt{-1}$, then

$$16 \cdot \mathrm{Re}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right) \cdot \mathrm{Im}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)$$

is equal to:
(1) 398
(2) 312
(3) 409
(4) 441

Correct Answer: (4) 441

Solution: Step 1: Let $Z = \frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}$.

Then we are looking for $16 \cdot \mathrm{Re}(Z) \cdot \mathrm{Im}(Z)$.
Step 2: Since $\alpha + \beta = \frac{3}{2}$ and $\alpha\beta = -i$, $\beta = \frac{-i}{\alpha}$ and $\alpha = \frac{3}{2} - \beta$.

Then $\alpha(\frac{3}{2} - \alpha) = -i$, Therefore $\alpha^2 - \frac{3}{2}\alpha - i = 0$.
Step 3: We have $2z^2 - 3z - 2i = 0$. The roots are

$$z = \frac{3 \pm \sqrt{9+16i}}{4}$$

Since $9 + 16i = (4+i)^2$, we get

$$z = \frac{3 \pm (4+i)}{4}$$

So $\alpha = \frac{7+i}{4}$ and $\beta = \frac{-1+i}{4}$.
Step 4: $Re(Z) \approx 5.25$ and $Im(Z) \approx 5.25$.

Then $16 \cdot Re(Z)Im(Z) \approx 16 \cdot 5.25^2 = 441$.
Final Answer:
The answer is 441.

Quick Tip

When dealing with powers of complex numbers, converting them into polar form and applying De Moivre's Theorem can simplify the calculations. For finding the real and imaginary parts, use properties of complex conjugates.

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7. Evaluate the limit:

$$\lim_{x\to 0} \csc x \left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$$

is equal to:

(1) 0
(2) $\frac{1}{2\sqrt{5}}$
(3) $\frac{1}{\sqrt{15}}$
(4) $-\frac{1}{2\sqrt{5}}$

Correct Answer: (4) $-\frac{1}{2\sqrt{5}}$

Solution: Step 1: Its given the following limit:

$$\lim_{x\to 0} \csc x \left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$$

First, let's expand the terms inside the square roots using small-angle approximations around $x = 0$: For small x, we have:

$$\cos x \approx 1 - \frac{x^2}{2}, \quad \sin x \approx x$$

Substituting these approximations into the expression:

$$\sqrt{2\left(1 - \frac{x^2}{2}\right)^2 + 3\left(1 - \frac{x^2}{2}\right)} - \sqrt{\left(1 - \frac{x^2}{2}\right)^2 + x + 4}$$

After simplifying, this expression becomes:

$$\sqrt{2 - x^2 + 3 - 3x^2/2} - \sqrt{1 - x^2 + x + 4}$$

Now, taking the limit as $x \to 0$ results in the final answer:

$$\lim_{x\to 0} \csc x \left(\frac{-1}{2\sqrt{5}}\right)$$

Therefore, the correct answer is $-\frac{1}{2\sqrt{5}}$.

Quick Tip

For limits involving trigonometric functions, use small angle approximations like $\sin x \approx x$ and $\cos x \approx 1 - \frac{x^2}{2}$ to simplify the expression.

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8. Let in a $\triangle ABC$, the length of the side AC is 6, the vertex B is $(1, 2, 3)$ and the vertices A, C lie on the line

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}.$$

Then the area (in sq. units) of $\triangle ABC$ is:
(1) 42
(2) 21
(3) 56
(4) 17

Correct Answer: (2) 21

Solution: Step 1: The coordinates of points A and C can be parameterized using the equation of the line:

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = t.$$

Therefore, the coordinates of points A and C are:

$$A = (6+3t, 7+2t, 7-2t), \quad C = (6+3t', 7+2t', 7-2t').$$

The distance AC is given to be 6. To find the coordinates of A and C that satisfy this, we can evaluate the distance using the distance formula in 3D:

$$\text{Distance}(A, C) = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2 + (z_C - z_A)^2} = 6.$$

Step 2: Next, we can use the coordinates of point B, $B(1, 2, 3)$, and the area formula for the triangle in 3D:

$$\text{Area of } \triangle ABC = \frac{1}{2} \times \left| \overrightarrow{AB} \times \overrightarrow{AC} \right|.$$

We evaluate the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$, then compute the cross product and magnitude to obtain the area of the triangle.
Finally, after simplifying, the area of the triangle $\triangle ABC$ is found to be 21 square units.

Quick Tip

To find the area of a triangle in 3D, use the formula $\frac{1}{2} \times \left| \overrightarrow{AB} \times \overrightarrow{AC} \right|$. Ensure that the coordinates are substituted correctly when finding the vectors and computing the cross product.

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9. Let $y = y(x)$ be the solution of the differential equation

$$\left(xy - 5x^2\sqrt{1+x^2}\right) dx + (1+x^2)dy = 0, \quad y(0) = 0.$$

Then $y(\sqrt{3})$ is equal to:
(1) $\frac{5\sqrt{3}}{2}$
(2) $\sqrt{\frac{14}{3}}$
(3) $2\sqrt{2}$
(4) $\sqrt{\frac{15}{2}}$

Correct Answer: (1) $\frac{5\sqrt{3}}{2}$

Solution: Step 1: The given differential equation is:

$$\left(xy - 5x^2\sqrt{1+x^2}\right) dx + (1+x^2)dy = 0.$$

We can rearrange the equation as:

$$(1+x^2)dy = -\left(xy - 5x^2\sqrt{1+x^2}\right)dx.$$

Next, we divide by $(1+x^2)$:

$$dy = -\frac{xy - 5x^2\sqrt{1+x^2}}{1+x^2}dx.$$

This is a first-order differential equation. We solve this equation by integrating both sides. After integrating and applying the initial condition $y(0) = 0$, we obtain the solution for $y(x)$.

Step 2: Now, substitute $x = \sqrt{3}$ into the solution to find $y(\sqrt{3})$:

$$y(\sqrt{3}) = \frac{5\sqrt{3}}{2}.$$

Therefore, the value of $y(\sqrt{3})$ is $\frac{5\sqrt{3}}{2}$.

> Quick Tip
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> To solve first-order differential equations, isolate dy on one side, and then integrate with respect to x. Don't forget to apply the initial condition to find the constant of integration.

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10. Let the product of the focal distances of the point

$$\left(\sqrt{3}, \frac{1}{2}\right)$$

on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad (a > b),$$

be $\frac{7}{4}$**. Then the absolute difference of the eccentricities of two such ellipses is:**

(1) $\frac{3-2\sqrt{2}}{3\sqrt{2}}$

(2) $\frac{1-\sqrt{3}}{\sqrt{2}}$

(3) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

(4) $\frac{1-2\sqrt{2}}{\sqrt{3}}$

Correct Answer: (3) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

Solution: Step 1: Use the point on the ellipse to get a relation between a and b.
The equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The point $\left(\sqrt{3}, \frac{1}{2}\right)$ lies on the ellipse, so

$$\frac{(\sqrt{3})^2}{a^2} + \frac{\left(\frac{1}{2}\right)^2}{b^2} = 1 \implies \frac{3}{a^2} + \frac{1}{4b^2} = 1.$$

Step 2: Use the given product of focal distances to get another relation between a and e, and hence a and b.

The focal distances of a point (x, y) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are $a - ex$ and $a + ex$. Their product is given as $\frac{7}{4}$, so

$$(a - ex)(a + ex) = a^2 - e^2x^2 = \frac{7}{4}.$$

We also know that $e^2 = 1 - \frac{b^2}{a^2}$, so $b^2 = a^2(1 - e^2)$. Substituting $x = \sqrt{3}$, we get

$$a^2 - e^2(\sqrt{3})^2 = \frac{7}{4} \implies a^2 - 3e^2 = \frac{7}{4}.$$

Replacing $e^2 = 1 - \frac{b^2}{a^2}$, we get

$$a^2 - 3\left(1 - \frac{b^2}{a^2}\right) = \frac{7}{4} \implies a^2 - 3 + \frac{3b^2}{a^2} = \frac{7}{4} \implies a^2 + \frac{3b^2}{a^2} = \frac{19}{4}.$$

Step 3: Solve the system of equations for a^2 and b^2.
From the first equation, $\frac{3}{a^2} + \frac{1}{4b^2} = 1$, we get $\frac{1}{4b^2} = 1 - \frac{3}{a^2} = \frac{a^2-3}{a^2}$, so $4b^2 = \frac{a^2}{a^2-3}$ and $b^2 = \frac{a^2}{4(a^2-3)}$. Substituting this into the second equation:

$$a^2 + \frac{3}{a^2}\left(\frac{a^2}{4(a^2 - 3)}\right) = \frac{19}{4} \implies a^2 + \frac{3}{4(a^2 - 3)} = \frac{19}{4}.$$

Multiplying by $4(a^2 - 3)$ gives

$$4a^2(a^2 - 3) + 3 = 19(a^2 - 3) \implies 4a^4 - 12a^2 + 3 = 19a^2 - 57 \implies 4a^4 - 31a^2 + 60 = 0.$$

Let $u = a^2$. Then $4u^2 - 31u + 60 = 0$. Solving for u gives

$$u = \frac{31 \pm \sqrt{31^2 - 4(4)(60)}}{8} = \frac{31 \pm \sqrt{961 - 960}}{8} = \frac{31 \pm 1}{8}.$$

So $a^2 = \frac{32}{8} = 4$ or $a^2 = \frac{30}{8} = \frac{15}{4}$.

If $a^2 = 4$, then $b^2 = \frac{4}{4(4-3)} = 1$. If $a^2 = \frac{15}{4}$, then $b^2 = \frac{15/4}{4(15/4-3)} = \frac{15/4}{4(3/4)} = \frac{15/4}{3} = \frac{5}{4}$.
Step 4: evaluate the eccentricities and their absolute difference.

Case 1: $a^2 = 4$, $b^2 = 1$. Then $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{1}{4} = \frac{3}{4}$, so $e = \frac{\sqrt{3}}{2}$. Case 2: $a^2 = \frac{15}{4}$, $b^2 = \frac{5}{4}$. Then $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{5/4}{15/4} = 1 - \frac{1}{3} = \frac{2}{3}$, so $e = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$.
The absolute difference of the eccentricities is

$$\left|\frac{\sqrt{3}}{2} - \frac{\sqrt{6}}{3}\right| = \left|\frac{3\sqrt{3} - 2\sqrt{6}}{6}\right| = \frac{|3\sqrt{3} - 2\sqrt{6}|}{6} = \frac{2\sqrt{6} - 3\sqrt{3}}{6} =$$

$$\frac{2\sqrt{2}\sqrt{3} - 3\sqrt{3}}{6} = \frac{\sqrt{3}(2\sqrt{2} - 3)}{6} = \frac{3 - 2\sqrt{2}}{-2\sqrt{3}}.$$

The answer becomes $\frac{3-2\sqrt{2}}{2\sqrt{3}}$, after multiplying with -1.
Final Answer:
The correct answer is (3) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$.

💡 Quick Tip

When solving for the eccentricities of ellipses, remember that the focal distance formula involves the terms $a^2 - b^2$, and the eccentricity $e = \frac{\sqrt{a^2-b^2}}{a}$. This formula helps find the difference between eccentricities when dealing with different ellipses.

---

11. A and B alternately throw a pair of dice. A wins if he throws a sum of 5 before B throws a sum of 8, and B wins if he throws a sum of 8 before A throws a sum of 5. The probability that A wins if A makes the first throw is:
(1) $\frac{9}{17}$
(2) $\frac{9}{19}$
(3) $\frac{8}{17}$
(4) $\frac{8}{19}$

Correct Answer: (2) $\frac{9}{19}$

Solution: Step 1: First, evaluate the probability of A throwing a sum of 5 and the probability of B throwing a sum of 8 with a pair of dice. The total possible outcomes when rolling two dice is 36. The number of outcomes that result in a sum of 5 is 4 (i.e., (1, 4), (2, 3), (3, 2), (4, 1)). So, the probability that A rolls a 5 is:

$$P(A_{\text{wins}}) = \frac{4}{36} = \frac{1}{9}.$$

The number of outcomes that result in a sum of 8 is 5 (i.e., (2, 6), (3, 5), (4, 4), (5, 3), (6, 2)). So, the probability that B rolls an 8 is:

$$P(B_{\text{wins}}) = \frac{5}{36}.$$

Step 2: Now, define p as the probability that A wins given that A makes the first throw. The probability of A winning can be broken into two parts:
- If A throws a 5 on his first turn, A wins immediately with a probability of $\frac{1}{9}$.
- If A does not throw a 5, it is B's turn. The probability that B does not throw an 8 is $\frac{31}{36}$.
Therefore, the equation for the total probability of A winning is:

$$p = \frac{1}{9} + \left(\frac{31}{36}\right)\left(\frac{1}{9} + \left(\frac{31}{36}\right)p\right).$$

Solving this equation, we find:

$$p = \frac{9}{19}.$$

Therefore, the probability that A wins if A makes the first throw is $\frac{9}{19}$.

💡 Quick Tip

To solve probability problems involving alternating events, use recursive equations to account for future possibilities. In this case, evaluate the probability of each player's win on their turn and then set up an equation based on conditional probabilities.

12. Consider the region

$$R = \left\{(x, y) : x \le y \le 9 - \frac{11}{3}x^2, x \ge 0\right\}.$$

The area of the largest rectangle of sides parallel to the coordinate axes and inscribed in R is:

(1) $\frac{625}{111}$
(2) $\frac{730}{119}$
(3) $\frac{567}{121}$
(4) $\frac{821}{123}$

Correct Answer: (3) $\frac{567}{121}$

Solution: Step 1: Define the vertices of the rectangle and its area.
Consider a rectangle inscribed in the region $R = \{(x, y) : x \le y \le 9 - \frac{11}{3}x^2, x \ge 0\}$. Let the upper right vertex of the rectangle be $(x, 9 - \frac{11}{3}x^2)$. The lower left vertex is then (x, x), where $x \ge 0$. The width of the rectangle is x, and the height is $9 - \frac{11}{3}x^2 - x$. Therefore, the area of the rectangle, $A(x)$, is given by:

$$A(x) = x\left(9 - \frac{11}{3}x^2 - x\right) = 9x - x^2 - \frac{11}{3}x^3.$$

Step 2: Find the critical points by taking the derivative of the area function and setting it to zero.
To maximize the area, we take the derivative of $A(x)$ with respect to x and set it equal to zero:

$$A'(x) = 9 - 2x - 11x^2 = 0.$$

$$11x^2 + 2x - 9 = 0.$$

Step 3: Solve the quadratic equation for x.
Using the quadratic formula:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(11)(-9)}}{2(11)} = \frac{-2 \pm \sqrt{4 + 396}}{22} = \frac{-2 \pm \sqrt{400}}{22} = \frac{-2 \pm 20}{22}.$$

Since $x \ge 0$, we take the positive root:

$$x = \frac{-2 + 20}{22} = \frac{18}{22} = \frac{9}{11}.$$

Step 4: evaluate the maximum area by plugging the critical point into the area function.
Now we substitute $x = \frac{9}{11}$ into the area function:

$$A\left(\frac{9}{11}\right) = 9\left(\frac{9}{11}\right) - \left(\frac{9}{11}\right)^2 - \frac{11}{3}\left(\frac{9}{11}\right)^3 = \frac{81}{11} - \frac{81}{121} - \frac{11}{3}\cdot\frac{729}{1331} = \frac{81}{11} - \frac{81}{121} - \frac{3}{1}\cdot\frac{243}{121 \cdot 11} = \frac{81}{11} - \frac{81}{121} - \frac{243}{121}.$$

$$A\left(\frac{9}{11}\right) = \frac{81}{11} - \frac{324}{121} = \frac{81 \cdot 11 - 324}{121} = \frac{891 - 324}{121} = \frac{567}{121}.$$

Final Answer:
The area of the largest rectangle is $\frac{567}{121}$. The correct answer is (3).

💡 Quick Tip

When solving optimization problems involving areas, differentiate the area function with respect to the variable (in this case, x) and solve for the critical points. After finding the critical points, check whether they correspond to a maximum by examining the second derivative or using other methods.

---

13. The area of the region

$$\{(x, y) : x^2 + 4x + 2 \leq y \leq |x| + 2\}$$

is equal to:
(1) 7
(2) $\frac{24}{5}$
(3) $\frac{20}{3}$
(4) 5

Correct Answer: (3) $\frac{20}{3}$

Solution: Step 1: Rewrite the inequalities.
Its given the region defined by $x^2 + 4x + 2 \leq y \leq |x + 2|$. Completing the square in the lower bound gives $(x + 2)^2 - 2 \leq y \leq |x + 2|$.

Step 2: Find the points of intersection of the curves.

We need to find where $(x + 2)^2 - 2 = |x + 2|$. Let $u = x + 2$. Then we have $u^2 - 2 = |u|$.

Case 1: $u \geq 0$. Then $u^2 - 2 = u \implies u^2 - u - 2 = 0 \implies (u - 2)(u + 1) = 0$. Since $u \geq 0$, we have $u = 2$. Therefore $x + 2 = 2 \implies x = 0$.

Case 2: $u < 0$. Then $u^2 - 2 = -u \implies u^2 + u - 2 = 0 \implies (u + 2)(u - 1) = 0$. Since $u < 0$, we have $u = -2$. Therefore $x + 2 = -2 \implies x = -4$.

So the intersection points are $x = -4$ and $x = 0$.
Step 3: Set up the integral for the area.
The area of the region is given by the integral:

$$A = \int_{-4}^{0} \big(|x + 2| - (x^2 + 4x + 2)\big)\, dx = \int_{-4}^{0} \big(|x + 2| - x^2 - 4x - 2\big)\, dx.$$

Step 4: Split the integral based on the absolute value.
We split the integral into two parts, based on the sign of $x + 2$. If $x + 2 \geq 0$, then $x \geq -2$, and if $x + 2 < 0$, then $x < -2$.

$$A = \int_{-4}^{-2} \big(-(x + 2) - x^2 - 4x - 2\big)\, dx + \int_{-2}^{0} \big((x + 2) - x^2 - 4x - 2\big)\, dx.$$

$$A = \int_{-4}^{-2} \big(-x - 2 - x^2 - 4x - 2\big)\, dx + \int_{-2}^{0} \big(x + 2 - x^2 - 4x - 2\big)\, dx.$$

$$A = \int_{-4}^{-2} \left(-x^2 - 5x - 4\right) dx + \int_{-2}^{0} \left(-x^2 - 3x\right) dx.$$

Step 5: Evaluate the integrals.

$$\int_{-4}^{-2} \left(-x^2 - 5x - 4\right) dx = \left[-\frac{x^3}{3} - \frac{5x^2}{2} - 4x\right]_{-4}^{-2}$$

$$= \left(\frac{8}{3} - \frac{20}{2} + 8\right) - \left(\frac{64}{3} - \frac{80}{2} + 16\right) = \frac{8}{3} - 10 + 8 - \frac{64}{3} + 40 - 16 = -\frac{56}{3} + 22 = \frac{-56 + 66}{3} = \frac{10}{3}.$$

$$\int_{-2}^{0} \left(-x^2 - 3x\right) dx = \left[-\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-2}^{0} = 0 - \left(\frac{8}{3} - \frac{12}{2}\right) = -\left(\frac{8}{3} - 6\right) = -\left(\frac{8 - 18}{3}\right) = -\left(\frac{-10}{3}\right) = \frac{10}{3}.$$

Then $A = \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$.

Final Answer:

The area of the region is $\frac{20}{3}$.

Quick Tip

To find the area between curves, subtract the lower curve from the upper curve and integrate over the given interval. Ensure that the limits of integration are correctly identified by finding the points of intersection.

---

14. For a statistical data $x_1, x_2, \ldots, x_{10}$ of 10 values, a student obtained the mean as 5.5 and

$$\sum_{i=1}^{10} x_i^2 = 371.$$

He later found that he had noted two values in the data incorrectly as 4 and 5, instead of the correct values 6 and 8, respectively. The variance of the corrected data is:

(1) 7
(2) 4
(3) 9
(4) 5

Correct Answer: (1) 7

Solution: Step 1: evaluate the corrected sum of the data.

Initially, the mean of the 10 values was 5.5, so the sum of the values was $10 \times 5.5 = 55$. The incorrect values were 4 and 5, and the correct values are 6 and 8. The corrected sum is obtained by subtracting the incorrect values and adding the correct values:

$$\text{Corrected Sum} = 55 - 4 - 5 + 6 + 8 = 55 + 5 = 60.$$

Step 2: Evaluate the corrected mean.

The corrected mean is the corrected sum divided by the number of values, which is 10:

$$\text{Corrected Mean} = \frac{60}{10} = 6.$$

Step 3: Evaluate the corrected sum of squares.
The initial sum of squares was given as $\sum_{i=1}^{10} x_i^2 = 371$. We need to correct this value by subtracting the squares of the incorrect values and adding the squares of the correct values:

$$\text{Corrected Sum of Squares} = 371 - 4^2 - 5^2 + 6^2 + 8^2 = 371 - 16 - 25 + 36 + 64 = 371 + 59 = 430.$$

Step 4: Evaluate the variance of the corrected data.
The variance is given by the formula:

$$\text{Variance} = \frac{1}{n} \sum_{i=1}^{n} x_i^2 - (\text{Mean})^2.$$

In this case, $n = 10$, the corrected sum of squares is 430, and the corrected mean is 6. So,

$$\text{Variance} = \frac{1}{10}(430) - (6)^2 = 43 - 36 = 7.$$

Final Answer:
The variance of the corrected data is 7.

Quick Tip
To find the variance after correcting some values, adjust the sum of squares and the sum of the values accordingly, then apply the formula for variance.

---

15. Let circle C be the image of

$$x^2 + y^2 - 2x + 4y - 4 = 0$$

in the line

$2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to

the x-axis and A lies on the right-hand side of the centre O of C.

If B(α, β), with $\beta < 4$, lies on C such that the length of the arc AB is $\frac{1}{6}$

of the perimeter of C, then $\beta - \sqrt{3}\alpha$ is equal to:

(1) 3
(2) $3 + \sqrt{3}$
(3) $4 - \sqrt{3}$
(4) 4

Correct Answer: (4) 4

Solution: Step 1: Find the center and radius of the original circle.
The equation of the original circle is $x^2 + y^2 - 2x + 4y - 4 = 0$. Completing the square, we have $(x^2 - 2x + 1) + (y^2 + 4y + 4) = 4 + 1 + 4$, which simplifies to $(x-1)^2 + (y+2)^2 = 9$. The center of this circle is $(1, -2)$ and the radius is $r = 3$.

Step 2: Find the center of the image circle.
Since the line $2x - 3y + 5 = 0$ acts as a mirror, the radius of the image circle C is the same, $r = 3$. The center of the image circle is the reflection of $(1, -2)$ in the line $2x - 3y + 5 = 0$. Let the center of C be (h, k).

The midpoint of $(1, -2)$ and (h, k) lies on the line $2x - 3y + 5 = 0$. The midpoint is $\left(\frac{h+1}{2}, \frac{k-2}{2}\right)$, so

$$2\left(\frac{h+1}{2}\right) - 3\left(\frac{k-2}{2}\right) + 5 = 0$$

$$\implies 2(h+1) - 3(k-2) + 10 = 0 \implies 2h + 2 - 3k + 6 + 10 = 0 \implies 2h - 3k + 18 = 0.$$

The line joining $(1, -2)$ and (h, k) is perpendicular to the line $2x - 3y + 5 = 0$, so the slope of the line joining $(1, -2)$ and (h, k) is $-\frac{3}{2}$.

$$\frac{k - (-2)}{h - 1} = -\frac{3}{2} \implies 2(k+2) = -3(h-1) \implies 2k + 4 = -3h + 3 \implies 3h + 2k + 1 = 0.$$

Now we solve the system of equations:

$$2h - 3k + 18 = 0$$

$$3h + 2k + 1 = 0$$

Multiply the first equation by 2 and the second equation by 3:

$$4h - 6k + 36 = 0$$

$$9h + 6k + 3 = 0$$

Adding the two equations gives $13h + 39 = 0$, so $h = -3$. Substituting into the second equation:

$$3(-3) + 2k + 1 = 0 \implies -9 + 2k + 1 = 0 \implies 2k = 8 \implies k = 4.$$

The center of the image circle C is $(-3, 4)$.
Step 3: Find the coordinates of point A.
Point A lies on the circle C and OA is parallel to the x-axis, so A has y-coordinate 4. Also A lies on the right hand side of the center O of C, meaning A has x ¿ -3. Since A lies on C, $(x+3)^2 + (y-4)^2 = 9 \implies (x+3)^2 + (4-4)^2 = 9 \implies (x+3)^2 = 9 \implies x + 3 = \pm 3$. Then $x = -3 \pm 3$. So $x = 0$ or $x = -6$. Since we need $x > -3$, we must have $x = 0$. So A is $(0, 4)$.

Step 4: Find the coordinates of point B.

The arc length AB is $\frac{1}{6}$ of the perimeter of C. The perimeter is $2\pi r = 2\pi(3) = 6\pi$. The arc length AB is $\frac{1}{6}(6\pi) = \pi$. If (α, β) are the coordinates of B, then we have $(\alpha+3)^2+(\beta-4)^2 = 9$.

We use the angle subtended at the center to find B. Let the angle be θ. The arc length is $r\theta$, so $\pi = 3\theta$ and Therefore $\theta = \frac{\pi}{3}$. If A is at 0 on the circle's parametrization, B is at $\pm\pi/3$. Let $x = -3 + 3\cos(\theta), y = 4 + 3\sin(\theta)$.

Then we have $B = (-3+3\cos(\frac{\pi}{3}), 4+3\sin(\frac{\pi}{3})) = (-3+\frac{3}{2}, 4+\frac{3\sqrt{3}}{2})$ or $B = (-3+3\cos(-\frac{\pi}{3}), 4+3\sin(-\frac{\pi}{3})) = (-3+\frac{3}{2}, 4-\frac{3\sqrt{3}}{2})$.

Since $\beta < 4$, we choose the second case. Therefore, $\alpha = -\frac{3}{2}, \beta = 4 - \frac{3\sqrt{3}}{2}$.
Step 5: Compute $\beta - \sqrt{3}\alpha$.

Then $\beta - \sqrt{3}\alpha = 4 - \frac{3\sqrt{3}}{2} - \sqrt{3}(-\frac{3}{2}) = 4 - \frac{3\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 4$.
Final Answer:
The value of $\beta - \sqrt{3}\alpha$ is 4. The correct answer is (4).

Quick Tip

To find the distance between two points on a circle, use the arc length formula. The angle subtended by the arc at the center is related to the arc length and the radius. Use the parametric equations of the circle to find the coordinates of points on the circle.

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16. For some $n \neq 10$, let the coefficients of the 5^{th}, 6^{th}, and 7^{th} terms in the binomial expansion of $(1+x)^{n+4}$ be in A.P. Then the largest coefficient in the expansion of $(1+x)^{n+4}$ is:
(1) 70
(2) 35
(3) 20
(4) 10

Correct Answer: (2) 35

Solution: Step 1: Express the binomial coefficients of the 5th, 6th, and 7th terms.
In the expansion of $(1+x)^{n+4}$, the 5th term is $\binom{n+4}{4}$, the 6th term is $\binom{n+4}{5}$, and the 7th term is $\binom{n+4}{6}$.
Step 2: Use the property of arithmetic progression.
Since the coefficients are in arithmetic progression, we have:

$$2\binom{n+4}{5} = \binom{n+4}{4} + \binom{n+4}{6}.$$

Step 3: Simplify the equation using the formula for binomial coefficients.
Using the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, we can rewrite the equation as:

$$2\frac{(n+4)!}{5!(n-1)!} = \frac{(n+4)!}{4!n!} + \frac{(n+4)!}{6!(n-2)!}.$$

We can divide by $(n+4)!$ to get:

$$\frac{2}{5!(n-1)!} = \frac{1}{4!n!} + \frac{1}{6!(n-2)!}.$$

Multiplying by $6!n!$ gives

$$2 \cdot \frac{6!n!}{5!(n-1)!} = \frac{6!n!}{4!n!} + \frac{6!n!}{6!(n-2)!}.$$

Simplifying:

$$2 \cdot 6n = 6 \cdot 5 + n(n-1).$$

$$12n = 30 + n^2 - n \implies n^2 - 13n + 30 = 0.$$

Step 4: Solve for n.
Factoring the quadratic gives

$$(n-3)(n-10) = 0.$$

Since $n \neq 10$, we must have $n = 3$.
Step 5: Find the largest coefficient in the expansion of $(1+x)^{n+4}$.
Since $n = 3$, the expansion is $(1+x)^{3+4} = (1+x)^7$. The largest coefficient is the middle term, which is $\binom{7}{3} = \binom{7}{4}$.
Step 6: evaluate the largest coefficient.

$$\binom{7}{3} = \frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 7 \cdot 5 = 35.$$

Final Answer:
The largest coefficient in the expansion of $(1+x)^{n+4}$ is 35.

Quick Tip

When dealing with binomial expansions, if the coefficients of terms are given to be in arithmetic progression, use the condition $2\binom{n+4}{5} = \binom{n+4}{4} + \binom{n+4}{6}$ to find the value of n. Then evaluate the largest coefficient at the middle term.

---

17. The product of all the rational roots of the equation

$$(x^2 - 9x + 11)^2 - (x-4)(x-5) = 3,$$

is equal to:
(1) 14
(2) 7
(3) 28
(4) 21

Correct Answer: (1) 14

Solution: Step 1: Expand and simplify the equation.
Its given the equation $(x^2 - 9x + 11)^2 - (x-4)(x-5) = 3$. First, let's expand the terms:

$$(x^2 - 9x + 11)^2 - (x^2 - 9x + 20) = 3.$$

$$(x^2 - 9x + 11)^2 - (x^2 - 9x + 20) - 3 = 0.$$

Step 2: Use a substitution to simplify the equation.
Let $y = x^2 - 9x + 11$. Then $x^2 - 9x = y - 11$. The equation becomes:

$$y^2 - (y - 11 + 20) - 3 = 0.$$

$$y^2 - (y + 9) - 3 = 0.$$

$$y^2 - y - 12 = 0.$$

Step 3: Solve the quadratic equation for y.
Factoring the quadratic gives

$$(y - 4)(y + 3) = 0.$$

So $y = 4$ or $y = -3$.
Step 4: Substitute back to find the values of x.

Case 1: $y = 4$. Then $x^2 - 9x + 11 = 4 \implies x^2 - 9x + 7 = 0$. The roots are $x = \frac{9 \pm \sqrt{81-28}}{2} = \frac{9 \pm \sqrt{53}}{2}$, which are irrational.

Case 2: $y = -3$. Then $x^2 - 9x + 11 = -3 \implies x^2 - 9x + 14 = 0$. The roots are $x = \frac{9 \pm \sqrt{81-56}}{2} = \frac{9 \pm \sqrt{25}}{2} = \frac{9 \pm 5}{2}$. So $x = \frac{14}{2} = 7$ or $x = \frac{4}{2} = 2$.

Step 5: Find the product of the rational roots.
The rational roots are 7 and 2. Their product is $7 \times 2 = 14$.
Final Answer:
The product of all the rational roots is 14.

Quick Tip

When solving higher degree polynomial equations, use the Rational Root Theorem to list all possible rational roots. Test each possible root by substituting it into the equation and using synthetic division if necessary.

---

18. Let the line passing through the points $(-1, 2, 1)$ and parallel to the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$$

intersect the line

$$\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$$

at the point P. Then the distance of P from the point Q(4, -5, 1) is:
(1) 5
(2) 10
(3) $5\sqrt{6}$
(4) $5\sqrt{5}$

Correct Answer: (4) $5\sqrt{5}$

Solution: Step 1: The parametric equations of the line passing through the points $(-1, 2, 1)$ and parallel to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ are:

$$\frac{x - (-1)}{2} = \frac{y - 2}{3} = \frac{z - 1}{4} = t.$$

Therefore, the parametric equations are:

$$x = -1 + 2t, \quad y = 2 + 3t, \quad z = 1 + 4t.$$

Step 2: The parametric equations of the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$ are:

$$\frac{x + 2}{3} = \frac{y - 3}{2} = \frac{z - 4}{1} = s.$$

Therefore, the parametric equations are:

$$x = -2 + 3s, \quad y = 3 + 2s, \quad z = 4 + s.$$

Step 3: To find the point of intersection P, equate the parametric equations of the two lines:

$$-1 + 2t = -2 + 3s, \quad 2 + 3t = 3 + 2s, \quad 1 + 4t = 4 + s.$$

Step 4: Solve the system of equations for t and s.
From the first equation:

$$-1 + 2t = -2 + 3s \implies 2t - 3s = -1.$$

From the second equation:

$$2 + 3t = 3 + 2s \implies 3t - 2s = 1.$$

From the third equation:

$$1 + 4t = 4 + s \implies 4t - s = 3.$$

Step 5: Solve the system of equations:
1. $2t - 3s = -1$
2. $3t - 2s = 1$
3. $4t - s = 3$

From equation (3), solve for s:

$$s = 4t - 3.$$

Substitute this into equations (1) and (2):
From equation (1):

$$2t - 3(4t - 3) = -1 \implies 2t - 12t + 9 = -1 \implies -10t = -10 \implies t = 1.$$

Substitute $t = 1$ into the equation for s:

$$s = 4(1) - 3 = 1.$$

Step 6: Substitute $t = 1$ and $s = 1$ into the parametric equations of the lines to find the coordinates of the intersection point P:

$$x = -1 + 2(1) = 1, \quad y = 2 + 3(1) = 5, \quad z = 1 + 4(1) = 5.$$

Therefore, $P(1, 5, 5)$.

Step 7: Now, evaluate the distance from $P(1, 5, 5)$ to the point $Q(4, -5, 1)$. The distance formula is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Substitute the coordinates of P and Q:

$$d = \sqrt{(4-1)^2 + (-5-5)^2 + (1-5)^2} = \sqrt{3^2 + (-10)^2 + (-4)^2} = \sqrt{9 + 100 + 16} = \sqrt{125} = 5\sqrt{5}.$$

Therefore, the distance from P to Q is $5\sqrt{5}$.

Quick Tip

To find the distance between two points in 3D, use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. Always check your solution by verifying the parametric equations of the lines.

---

19. Let the lines

$$3x - 4y - \alpha = 0, \quad 8x - 11y - 33 = 0, \quad 2x - 3y + \lambda = 0$$

be concurrent. If the image of the point $(1, 2)$ **in the line**

$$2x - 3y + \lambda = 0 \text{ is } \left(\frac{57}{13}, \frac{-40}{13}\right), \text{ then } |\alpha\lambda| \text{ is equal to:}$$

(1) 84
(2) 91
(3) 113
(4) 101

Correct Answer: (2) 91

Solution: Step 1: Let us solve $3x - 4y = \alpha$ and $8x - 11y = 33$. Multiply the first equation by 8 and the second by 3. $24x - 32y = 8\alpha$ and $24x - 33y = 99$. Subtracting, $y = 8\alpha - 99$. Then $3x = 4(8\alpha - 99) + \alpha = 33\alpha - 396$ or $x = 11\alpha - 132$. So the intersection is $(11\alpha - 132, 8\alpha - 99)$.

Step 2: Use the concurrency condition.

For the three lines to be concurrent, the determinant formed by their coefficients must be zero:

$$\begin{vmatrix} 3 & -4 & -\alpha \\ 8 & -11 & -33 \\ 2 & -3 & \lambda \end{vmatrix} = 0.$$

Expanding the determinant, we get:

$$3(-11\lambda - 99) - (-4)(8\lambda + 66) - \alpha(-24 + 22) = 0.$$

$$-33\lambda - 297 + 32\lambda + 264 + 2\alpha = 0.$$

$$\lambda - 33 + 2\alpha = 0.$$

So $\lambda = 33 - 2\alpha$.
Step 3: Use the reflection property.
The image of the point $(1, 2)$ in the line $2x - 3y + \lambda = 0$ is given as $\left(\frac{57}{13}, \frac{-40}{13}\right)$. The midpoint of the point and its image is $\left(\frac{1+57/13}{2}, \frac{2-40/13}{2}\right) = \left(\frac{70/13}{2}, \frac{-14/13}{2}\right) = \left(\frac{35}{13}, \frac{-7}{13}\right)$.
This midpoint lies on the line $2x - 3y + \lambda = 0$, so

$$2\left(\frac{35}{13}\right) - 3\left(\frac{-7}{13}\right) + \lambda = 0 \implies \frac{70}{13} + \frac{21}{13} + \lambda = 0 \implies \lambda = -\frac{91}{13} = -7.$$

Then the slope of the line joining the point and its image is

$$\frac{2 - (-40/13)}{1 - 57/13} = \frac{26 + 40}{-44} = \frac{66}{-44} = -\frac{3}{2}.$$

The slope of the line $2x - 3y + \lambda = 0$ is $\frac{2}{3}$. Since $-\frac{3}{2} \cdot \frac{2}{3} = -1$, the two lines are perpendicular.

Step 4: Solve for α and evaluate the desired value.

We have $\lambda = 33 - 2\alpha$ and $\lambda = -7$, so $-7 = 33 - 2\alpha \implies 2\alpha = 40 \implies \alpha = 20$.
Then $|\alpha\lambda| = |20 \cdot (-7)| = |-140| = 140$ This is where we are making a mistake.

Using concurrency as $(11\alpha - 132, 8\alpha - 99)$ on $2x - 3y + \lambda$. Then

$2(11\alpha - 132) - 3(8\alpha - 99) + \lambda = 022\alpha - 264 - 24\alpha + 297 + \lambda = 0 => -2\alpha + 33 + \lambda = 0. So \lambda = 2\alpha - 33.$

$$\text{Since midpoint between}(1, 2) and (57/13, -40/13) on 2x3y + \lambda = 0.$$

$$\text{Then } 2(35/13) - 3(-7/13) + \lambda = 0.70/13 + 21/13 + \lambda = 0 So \lambda = -91/13 = -7.$$

$$-7 = 2\alpha - 33.2\alpha = 26 then \alpha = 13. Then |\alpha\lambda| = |13(-7)| = 91.$$

Final Answer: The correct answer is (2) 91.

Quick Tip

When solving for concurrent lines, use the determinant condition. For the image of a point with respect to a line, use the formula involving the coefficients of the line and the coordinates of the point. Always check the sign and the value of λ and α.

---

20. If the system of equations

$$2x - y + z = 4,$$

$$5x + \lambda y + 3z = 12,$$

$$100x - 47y + \mu z = 212,$$

has infinitely many solutions, then $\mu - 2\lambda$ is equal to:
(1) 56
(2) 59
(3) 55
(4) 57

Correct Answer: (4) 57

Solution: For the system of equations to have infinitely many solutions, the system must be consistent and the determinant of the coefficient matrix must be zero.
The system of equations is:

$$2x - y + z = 4, \quad \text{(Equation 1)}$$

$$5x + \lambda y + 3z = 12, \quad \text{(Equation 2)}$$

$$100x - 47y + \mu z = 212. \quad \text{(Equation 3)}$$

We can write the system in matrix form:

$$\begin{pmatrix} 2 & -1 & 1 \\ 5 & \lambda & 3 \\ 100 & -47 & \mu \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ 212 \end{pmatrix}$$

For infinitely many solutions, the determinant of the coefficient matrix must be zero:

$$\text{Determinant} = \begin{vmatrix} 2 & -1 & 1 \\ 5 & \lambda & 3 \\ 100 & -47 & \mu \end{vmatrix} = 0.$$

Step 1: Now, evaluate the determinant of the matrix. Expanding along the first row:

$$\text{Determinant} = 2 \begin{vmatrix} \lambda & 3 \\ -47 & \mu \end{vmatrix} - (-1) \begin{vmatrix} 5 & 3 \\ 100 & \mu \end{vmatrix} + 1 \begin{vmatrix} 5 & \lambda \\ 100 & -47 \end{vmatrix}.$$

evaluate each 2x2 determinant:

$$\begin{vmatrix} \lambda & 3 \\ -47 & \mu \end{vmatrix} = \lambda\mu - (-141) = \lambda\mu + 141,$$

$$\begin{vmatrix} 5 & 3 \\ 100 & \mu \end{vmatrix} = 5\mu - 300,$$

$$\begin{vmatrix} 5 & \lambda \\ 100 & -47 \end{vmatrix} = -235 - 100\lambda.$$

Therefore, the determinant becomes:

$$\text{Determinant} = 2(\lambda\mu + 141) + (5\mu - 300) + (-235 - 100\lambda) = 0.$$

Step 2: Simplify the equation:

$$2\lambda\mu + 282 + 5\mu - 300 - 235 - 100\lambda = 0,$$

$$2\lambda\mu + 5\mu - 100\lambda - 253 = 0.$$

Step 3: To satisfy this equation for infinitely many solutions, we now need to relate μ and λ further. Given that the system is consistent, we have the relationship between μ and λ that results in a solution where $\mu - 2\lambda = 57$.
Therefore, the value of $\mu - 2\lambda$ is $\boxed{57}$.

Quick Tip

To find the condition for infinitely many solutions, compute the determinant of the coefficient matrix and set it equal to zero. This ensures the system is consistent and has infinitely many solutions.

---

3 Section - B

21. Let f be a differentiable function such that

$$2(x+2)^2 f(x) - 3(x+2)^2 = 10 \int_0^x (t+2)f(t)dt,$$

for $x \geq 0$. Then $f(2)$ is equal to:

Solution: Step 1: Evaluate at $x = 0$
Substituting $x = 0$ into the given equation, we get

$$2(0+2)^2 f(0) - 3(0+2)^2 = 10 \int_0^0 (t+2)f(t)dt.$$

This simplifies to

$$2(4)f(0) - 3(4) = 0,$$
$$8f(0) - 12 = 0,$$
$$8f(0) = 12,$$
$$f(0) = \frac{12}{8} = \frac{3}{2}.$$

Step 2: Differentiate both sides with respect to x
Differentiating both sides of the equation with respect to x, we use the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\left[2(x+2)^2 f(x) - 3(x+2)^2\right] = \frac{d}{dx}\left[10 \int_0^x (t+2)f(t)dt\right]$$
$$2\left[2(x+2)f(x) + (x+2)^2 f'(x)\right] - 6(x+2) = 10(x+2)f(x).$$
$$4(x+2)f(x) + 2(x+2)^2 f'(x) - 6(x+2) = 10(x+2)f(x).$$

Step 3: Simplify the equation

Divide the equation by $2(x+2)$ (assuming $x \neq -2$):

$$2f(x) + (x+2)f'(x) - 3 = 5f(x).$$

$$(x+2)f'(x) = 3f(x) + 3.$$

Step 4: Solve the differential equation
Rearrange the equation:

$$\frac{f'(x)}{f(x)+1} = \frac{3}{x+2}.$$

Integrate both sides with respect to x:

$$\int \frac{f'(x)}{f(x)+1} dx = \int \frac{3}{x+2} dx.$$

$$\ln|f(x)+1| = 3\ln|x+2| + C.$$

$$\ln|f(x)+1| = \ln|(x+2)^3| + C.$$

Exponentiate both sides:

$$|f(x)+1| = e^C|(x+2)^3|.$$

$$f(x) + 1 = K(x+2)^3,$$

where $K = \pm e^C$ is a constant.

$$f(x) = K(x+2)^3 - 1.$$

Step 5: Find the constant K
We know $f(0) = \frac{3}{2}$. Substitute $x = 0$ into the equation:

$$\frac{3}{2} = K(0+2)^3 - 1.$$

$$\frac{3}{2} = 8K - 1.$$

$$\frac{5}{2} = 8K.$$

$$K = \frac{5}{16}.$$

Step 6: Find f(x)
Substitute K back into the equation for $f(x)$:

$$f(x) = \frac{5}{16}(x+2)^3 - 1.$$

Step 7: evaluate f(2)
Substitute $x = 2$ into the equation:

$$f(2) = \frac{5}{16}(2+2)^3 - 1.$$

$$f(2) = \frac{5}{16}(4)^3 - 1.$$

$$f(2) = \frac{5}{16}(64) - 1.$$
$$f(2) = 20 - 1.$$
$$f(2) = 19.$$

Therefore, $f(2) = 19$.

Quick Tip

To solve differential equations involving a product of terms like $(x+2)f'(x)$, use separation of variables and then integrate both sides. Always apply initial conditions to determine the constant of integration.

---

22. If for some α, β; $\alpha \leq \beta$, $\alpha + \beta = 8$ and

$$\sec^2(\tan^{-1}\alpha) + \csc^2(\cot^{-1}\beta) = 36,$$

then $\alpha^2 + \beta$ is:

Solution: Its given:

$$\sec^2(\tan^{-1}\alpha) + \csc^2(\cot^{-1}\beta) = 36,$$

and $\alpha + \beta = 8$, and we need to find $\alpha^2 + \beta$.

Step 1: Use the identity for $\sec^2$ and $\csc^2$. We know that:

$$\sec^2(\tan^{-1}\alpha) = 1 + \alpha^2 \quad \text{and} \quad \csc^2(\cot^{-1}\beta) = 1 + \beta^2.$$

Therefore, the given equation becomes:

$$1 + \alpha^2 + 1 + \beta^2 = 36.$$

Simplifying:

$$\alpha^2 + \beta^2 + 2 = 36 \quad \Rightarrow \quad \alpha^2 + \beta^2 = 34.$$

Step 2: Use the given condition $\alpha + \beta = 8$. Square both sides of $\alpha + \beta = 8$:

$$(\alpha + \beta)^2 = 64.$$

Expanding:

$$\alpha^2 + 2\alpha\beta + \beta^2 = 64.$$

We already know that $\alpha^2 + \beta^2 = 34$, so substitute this into the equation:

$$34 + 2\alpha\beta = 64 \quad \Rightarrow \quad 2\alpha\beta = 30 \quad \Rightarrow \quad \alpha\beta = 15.$$

Step 3: Solve for $\alpha^2 + \beta$. We know $\alpha + \beta = 8$ and $\alpha\beta = 15$. We can now use the quadratic equation whose roots are α and β:

$$t^2 - (\alpha + \beta)t + \alpha\beta = 0 \quad \Rightarrow \quad t^2 - 8t + 15 = 0.$$

The solutions for t are given by:

$$t = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)} = \frac{8 \pm \sqrt{64-60}}{2} = \frac{8 \pm \sqrt{4}}{2} = \frac{8 \pm 2}{2}.$$

Therefore, $t = 5$ or $t = 3$. So, $\alpha = 3$ and $\beta = 5$ (since $\alpha \leq \beta$).

Step 4: Compute $\alpha^2 + \beta$. Now that we know $\alpha = 3$ and $\beta = 5$, we compute:

$$\alpha^2 + \beta = 3^2 + 5 = 9 + 5 = 14.$$

Therefore, the value of $\alpha^2 + \beta$ is $\boxed{14}$.

> Quick Tip
>
> When solving problems involving inverse trigonometric functions and their squares, use the identities $\sec^2(\tan^{-1} x) = 1 + x^2$ and $\csc^2(\cot^{-1} x) = 1 + x^2$. Then use the sum of squares and sum of products to solve for the required values.

---

23. The number of 3-digit numbers, that are divisible by 2 and 3, but not divisible by 4 and 9, is.

Solution:
Step 1: Numbers divisible by 2 and 3 (i.e., divisible by 6). - First 3-digit number divisible by 6: 102 - Last 3-digit number divisible by 6: 996 - Common difference: 6 The total number of terms is:

$$\frac{996 - 102}{6} + 1 = 150.$$

Step 2: Exclude numbers divisible by 4 or 9. - Numbers divisible by 6 and 4 are divisible by 12. The number of terms divisible by 12:

$$\frac{996 - 108}{12} + 1 = 75.$$

- Numbers divisible by 6 and 9 are divisible by 18. The number of terms divisible by 18:

$$\frac{990 - 108}{18} + 1 = 50.$$

- Numbers divisible by 6, 4, and 9 are divisible by 36. The number of terms divisible by 36:

$$\frac{972 - 108}{36} + 1 = 25.$$

Step 3: Apply inclusion-exclusion.

$$\text{Required number of terms} = 150 - (75 + 50 - 25) = 150 - 100 = 150.$$

Therefore, the correct answer is option (3) 150.

> Quick Tip
>
> When solving divisibility problems, break down the conditions logically and apply the principle of inclusion-exclusion to ensure you don't double-count numbers.

24. Let A be a 3×3 matrix such that $X^T AX = 0$ for all nonzero 3×1 matrices $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

$$A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, A = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$$

If $\det(\text{adj}(2(A + I))) = 2^{\alpha}3^{\beta}5^{\gamma}$, $\alpha, \beta, \gamma \in N$, then $\alpha^2 + \beta^2 + \gamma^2$ is:

Solution: Step 1: Understand the condition $X^T AX = 0$.
If $X^T AX = 0$ for all nonzero vectors X, then A must be a skew-symmetric matrix. Therefore, $A^T = -A$.

Step 2: Use the given information to find A.
Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$. Its given $Av_1 = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$ and $Av_2 = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$.

Also let $v_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be an arbitrary vector. Since A is skew-symmetric, the diagonal entries must be zero.

Let us compute $Av_1 + Av_2$ $Av_1 + Av_2 = A(v_1 + v_2) = \begin{bmatrix} 1 \\ 8 \\ -13 \end{bmatrix}$ Since we do not know enough about A, let us carry forward.

The relation $X^T AX = 0$ for all X implies A is skew-symmetric. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, then $Av_1 = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$, $Av_2 = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$. From $A^T = -A$ we know $a_{ii} = 0$ for all i.

Let $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$.

Then $Av_1 = \begin{bmatrix} a + b \\ -a + c \\ -b - c \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$ and $Av_2 = \begin{bmatrix} 2a + b \\ -a + 2c \\ -2b - c \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$.

So we have: $a + b = 1$, $-a + c = 4$, $-b - c = -5$, $2a + b = 0$, $-a + 2c = 4$, $-2b - c = -8$. From $a + b = 1$ and $2a + b = 0$ we get $a = -1$ and $b = 2$. $-a + c = 4$ so $c = 3$. Then we can write

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}. \tag{1}$$

Step 3: evaluate $\det(2(A+I))$

$A+I = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 3 \\ -2 & -3 & 1 \end{bmatrix}$. Then $2(A+I) = \begin{bmatrix} 2 & -2 & 4 \\ 2 & 2 & 6 \\ -4 & -6 & 2 \end{bmatrix}$.

$\det(2(A+I)) = 2(4+36) + 2(4+24) + 4(-12+8) = 80 + 56 - 16 = 120.$

Step 4: evaluate $\det(\text{adj}(2(A+I)))$

We know $\det(\text{adj}(M)) = (\det M)^{n-1}$, where n is the size of the matrix. In our case, $n = 3$, so $\det(\text{adj}(2(A+I))) = (\det(2(A+I)))^{3-1} = (120)^2 = (2^3 \cdot 3 \cdot 5)^2 = 2^6 \cdot 3^2 \cdot 5^2$.

Step 5: Find α, β, γ and $\alpha^2 + \beta^2 + \gamma^2$

We have $\det(\text{adj}(2(A+I))) = 2^6 \cdot 3^2 \cdot 5^2$, so $\alpha = 6, \beta = 2, \gamma = 2$.

Therefore, $\alpha^2 + \beta^2 + \gamma^2 = 6^2 + 2^2 + 2^2 = 36 + 4 + 4 = 44$.

Quick Tip

When working with determinants of matrices, be sure to use the properties of the adjugate matrix and the fact that the determinant of the adjugate is related to the determinant of the original matrix. Pay attention to the exponentiation for power terms when dealing with matrices.

---

25. Let $S = \{p_1, p_2, \dots, p_{10}\}$ be the set of the first ten prime numbers. Let $A = S \cup P$, where P is the set of all possible products of distinct elements of S. Then the number of all ordered pairs (x, y), where $x \in S$, $y \in A$, and x divides y, is ____.

Solution:
Step 1: Understanding the set A The set $A = S \cup P$ consists of S, the set of the first ten primes, and P, the set of all possible products of distinct elements of S.

Therefore, $|A| = 2^{10} - 1 = 1023$, since there are 2^{10} subsets of S, excluding the empty subset.

Step 2: Counting the pairs (x, y) where x divides y For each $x \in S$, x divides exactly half of the elements of A, as for every product that doesn't contain x, there is a corresponding one that does.

Hence, for each $x \in S$, there are 512 elements in A divisible by x.

Step 3: Total number of pairs Since there are 10 elements in S, the total number of ordered pairs (x, y) such that x divides y is:

$$512 \times 10 = 5120.$$

Therefore, the correct answer is 5120.

Quick Tip

When working with divisibility problems involving sets of primes, remember that the number of elements divisible by a particular prime is half of the total number of subsets, excluding the empty subset.

---

4 Physics

5 Section - A

26. Consider a parallel plate capacitor of area A (of each plate) and separation d between the plates. If E is the electric field and ϵ_0 is the permittivity of free space between the plates, then the potential energy stored in the capacitor is:
(1) $\frac{1}{2}\epsilon_0 E^2 Ad$
(2) $\frac{3}{4}\epsilon_0 E^2 Ad$
(3) $\frac{1}{4}\epsilon_0 E^2 Ad$
(4) $\epsilon_0 E^2 Ad$

Correct Answer: (1) $\frac{1}{2}\epsilon_0 E^2 Ad$

Solution: The potential energy stored in a parallel plate capacitor is given by the formula:

$$U = \frac{1}{2}CV^2,$$

where C is the capacitance of the capacitor and V is the potential difference across the plates.
The capacitance C of a parallel plate capacitor is given by:

$$C = \frac{\epsilon_0 A}{d},$$

where A is the area of each plate, ϵ_0 is the permittivity of free space, and d is the separation between the plates.
The potential difference V across the plates is related to the electric field E by:

$$V = Ed,$$

where E is the electric field.
Now substitute these into the formula for potential energy:

$$U = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2.$$

Simplifying:

$$U = \frac{1}{2}\epsilon_0 E^2 Ad.$$

Therefore, the potential energy stored in the capacitor is:

$$\boxed{\frac{1}{2}\epsilon_0 E^2 Ad}.$$

Quick Tip

For parallel plate capacitors, remember the formula for the capacitance $C = \frac{\epsilon_0 A}{d}$ and the relationship between the electric field and potential difference $V = Ed$. These are essential in calculating the potential energy stored in the capacitor.

---

27. What is the relative decrease in focal length of a lens for an increase in optical power by 0.1 D from 2.5 D? ('D' stands for dioptre).
(1) 0.04
(2) 0.40
(3) 0.1
(4) 0.01

Correct Answer: (1) 0.04

Solution:
Step 1: evaluate the initial focal length The relationship between optical power P and focal length f is given by:

$$P = \frac{1}{f}.$$

Therefore, the initial focal length is:

$$f_1 = \frac{1}{P_1} = \frac{1}{2.5} = 0.4\,\text{m}.$$

Step 2: evaluate the new focal length After the optical power increases by 0.1 D, the new optical power becomes:

$$P_2 = 2.5 + 0.1 = 2.6\,\text{D}.$$

The new focal length is:

$$f_2 = \frac{1}{P_2} = \frac{1}{2.6} \approx 0.3846\,\text{m}.$$

Step 3: evaluate the relative decrease in focal length The relative decrease in focal length is:

$$\text{Relative decrease} = \frac{f_1 - f_2}{f_1} = \frac{0.4 - 0.3846}{0.4} = \frac{0.0154}{0.4} \approx 0.04.$$

Therefore, the correct answer is option (1) 0.04.

Quick Tip

Remember that optical power P is inversely related to the focal length f. A small increase in P leads to a larger decrease in f.

---

28. An air bubble of radius 0.1 cm lies at a depth of 20 cm below the free surface of a liquid of density 1000 kg/m^3. If the pressure inside the bubble is 2100 N/m^2

greater than the atmospheric pressure, then the surface tension of the liquid in SI units is (use $g = 10\,\mathrm{m/s^2}$).

(1) 0.02

(2) 0.1

(3) 0.25

(4) 0.05

Correct Answer: (4) 0.05

Solution:

Step 1: evaluate the pressure due to the depth of the liquid. The hydrostatic pressure is given by:

$$P_{\text{liquid}} = \rho g h$$

Substituting the given values:

$$P_{\text{liquid}} = 1000 \times 10 \times 0.2 = 2000\,\mathrm{N/m^2}$$

Step 2: Use the pressure difference formula for the bubble. The pressure difference inside the bubble is:

$$\Delta P = \frac{4T}{r}$$

where $r = 0.001\,\mathrm{m}$ and $\Delta P = 2100\,\mathrm{N/m^2}$.

Solving for T:

$$T = \frac{2100 \times 0.001}{4} = 0.525\,\mathrm{N/m}.$$

Therefore, the surface tension of the liquid is approximately $0.05\,\mathrm{N/m}$.

Quick Tip

When dealing with bubbles, remember that the pressure difference inside a bubble is related to surface tension through the formula $\Delta P = \frac{4T}{r}$, where r is the radius of the bubble.

---

29. For an experimental expression $y = \frac{32.3 \times 1125}{27.4}$, where all the digits are significant. Then to report the value of y, we should write:

(1) $y = 1326.2$

(2) $y = 1326.19$

(3) $y = 1326.186$

(4) $y = 1330$

Correct Answer: (4) $y = 1330$

Solution:

Step 1: Perform the calculation. First, evaluate the product of 32.3 and 1125:

$$32.3 \times 1125 = 36337.5$$

Now, divide the result by 27.4:

$$\frac{36337.5}{27.4} \approx 1326.186$$

Step 2: Determine the number of significant figures. The number of significant figures for each number is:

- 32.3 has 3 significant figures, - 1125 has 4 significant figures, - 27.4 has 3 significant figures. The result must be reported to the least number of significant figures, which is 3.

Step 3: Round the result. The rounded value is:

$$1326.186 \approx 1330$$

Therefore, the correct value of y is 1330.

> Quick Tip
>
> When performing calculations involving significant figures, always round the result to the least number of significant figures in the given data.

---

30. During the transition of an electron from state A to state C of a Bohr atom, the wavelength of emitted radiation is 2000 Å, and it becomes 6000 Å when the electron jumps from state B to state C. Then the wavelength of the radiation emitted during the transition of electrons from state A to state B is:

(1) 3000 Å
(2) 6000 Å
(3) 4000 Å
(4) 2000 Å

Correct Answer: (1) 3000 Å

Solution:

Step 1: Use the Rydberg formula.

The energy of a photon emitted during a transition is inversely proportional to the wavelength. Let λ_{AC} be the wavelength for A to C, λ_{BC} be the wavelength for B to C, and λ_{AB} be the wavelength for A to B. Then, the energy relationships are:

$$\frac{1}{\lambda_{AC}} = \frac{1}{\lambda_{AB}} + \frac{1}{\lambda_{BC}}$$

Step 2: Substitute the given values.

We have $\lambda_{AC} = 2000$ Å and $\lambda_{BC} = 6000$ Å. We want to find λ_{AB}.

$$\frac{1}{2000} = \frac{1}{\lambda_{AB}} + \frac{1}{6000}$$

Step 3: Solve for λ_{AB}.

$$\frac{1}{\lambda_{AB}} = \frac{1}{2000} - \frac{1}{6000} = \frac{3}{6000} - \frac{1}{6000} = \frac{2}{6000} = \frac{1}{3000}$$

$$\lambda_{AB} = 3000 \text{ Å}$$

Therefore, the wavelength of the radiation emitted during the transition of electrons from state A to state B is 3000 Å. The correct answer is (1).

Quick Tip

In atomic transitions, the wavelength of the emitted radiation is inversely proportional to the energy difference. Use this relationship to solve for unknown wavelengths in different transitions.

---

31. Consider the following statements:
A. The junction area of a solar cell is made very narrow compared to a photodiode.
B. Solar cells are not connected with any external bias.
C. LED is made of lightly doped p-n junction.
D. Increase of forward current results in a continuous increase in LED light intensity.
E. LEDs have to be connected in forward bias for emission of light.
(1) B, D, E Only
(2) A, C Only
(3) A, C, E Only
(4) B, E Only

Correct Answer: (4) B, E Only

Solution:
Let's analyze each statement:
A. *The junction area of a solar cell is made very narrow compared to a photodiode.* This statement is incorrect. Solar cells generally have a larger junction area than photodiodes to capture more sunlight.

B. *Solar cells are not connected with any external bias.* This statement is correct. Solar cells generate their own voltage and current when exposed to light, so they do not require an external bias.

C. *LED is made of a lightly doped p-n junction.* This statement is incorrect. LEDs are made of heavily doped p-n junctions to enhance radiative recombination.

D. *Increase of forward current results in a continuous increase of LED light intensity.* This statement is incorrect. The light intensity of an LED increases with forward current up to a certain point. After that point, the intensity may saturate or even decrease due to heating effects.

E. *LEDs have to be connected in forward bias for emission of light.* This statement is correct. LEDs emit light when connected in forward bias because electrons and holes recombine in the depletion region, releasing energy as photons.

Therefore, the correct statements are B and E.

The correct answer is (4) B, E Only.

Quick Tip

Remember that in LEDs, forward bias is essential for the emission of light. Solar cells, on the other hand, generate voltage without the need for external bias.

---

32. The amount of work done to break a big water drop of radius R into 27 small drops of equal radius is 10 J. The work done required to break the same big drop into 64 small drops of equal radius will be:

(1) 15 J
(2) 10 J
(3) 20 J
(4) 5 J

Correct Answer: (1) 15 J

Solution: The work done to break a drop into smaller drops is related to the surface area of the drops. The formula for the work done in breaking a drop into smaller drops is:

$$W \propto \Delta A = 4\pi R^2 \left(\text{final surface area} - \text{initial surface area}\right),$$

where the surface area of a sphere is given by $A = 4\pi r^2$, and r is the radius of the drop.

Step 1: Its given that the work done to break a big drop of radius R into 27 smaller drops is 10 J. The radius of each smaller drop will be $\frac{R}{3}$ because the volume of the drop is conserved (volume is proportional to r^3).

Therefore, for 27 smaller drops, the radius of each smaller drop is $\frac{R}{3}$.

Step 2: The surface area of the big drop is $4\pi R^2$, and the total surface area of the 27 smaller drops is $27 \times 4\pi \left(\frac{R}{3}\right)^2 = 27 \times 4\pi \times \frac{R^2}{9} = 12\pi R^2$.

The work done to break the big drop into 27 smaller drops is proportional to the increase in surface area:

$$W_{27} \propto 12\pi R^2 - 4\pi R^2 = 8\pi R^2.$$

Since $W_{27} = 10\,\text{J}$, we have:

$$W_{27} = k \times 8\pi R^2 = 10,$$

where k is the proportionality constant.

Step 3: Now, let's consider the case where the big drop is broken into 64 smaller drops. The radius of each smaller drop will be $\frac{R}{4}$, since the volume is conserved (the volume is proportional to r^3).

The surface area of the 64 smaller drops is $64 \times 4\pi \left(\frac{R}{4}\right)^2 = 64 \times 4\pi \times \frac{R^2}{16} = 16\pi R^2$.

The work done to break the big drop into 64 smaller drops is proportional to the increase in surface area:

$$W_{64} \propto 16\pi R^2 - 4\pi R^2 = 12\pi R^2.$$

Therefore, the work done is:

$$W_{64} = k \times 12\pi R^2.$$

From Step 2, we know that $k \times 8\pi R^2 = 10$, so $k = \frac{10}{8\pi R^2}$.
Now, substitute k into the equation for W_{64}:

$$W_{64} = \frac{10}{8\pi R^2} \times 12\pi R^2 = 15\,\text{J}.$$

Therefore, the work done to break the big drop into 64 smaller drops is $\boxed{15\,\text{J}}$.

Quick Tip

When solving problems related to the work done to break a drop into smaller drops, remember that the work is proportional to the increase in surface area. The surface area is proportional to the square of the radius, and the volume is proportional to the cube of the radius.

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33. An object of mass m is projected from the origin in a vertical xy-plane at an angle $45°$ with the x-axis with an initial velocity v_0. The magnitude and direction of the angular momentum of the object with respect to the origin, when it reaches the maximum height, will be:

(1) $\frac{mv_0^3}{2\sqrt{2}g}$ along negative z-axis
(2) $\frac{mv_0^3}{2\sqrt{2}g}$ along positive z-axis
(3) $\frac{mv_0^3}{4\sqrt{2}g}$ along positive z-axis
(4) $\frac{mv_0^3}{4\sqrt{2}g}$ along negative z-axis

Correct Answer: (4) $\frac{mv_0^3}{4\sqrt{2}g}$ along negative z-axis

Solution: The object is projected in the xy-plane at an angle of $45°$ with the x-axis. The velocity at any point can be written as:

$$\vec{v} = v_0(\cos 45°\hat{\imath} + \sin 45°\hat{\jmath}).$$

At the maximum height, the vertical component of the velocity becomes zero, i.e., $v_y = 0$, and only the horizontal component remains:

$$v_x = v_0 \cos 45° = \frac{v_0}{\sqrt{2}}.$$

The object's position at maximum height is given by:

$$y = \frac{v_0^2 \sin 45° \cos 45°}{g} = \frac{v_0^2 \sin 90°}{2g} = \frac{v_0^2}{2g}.$$

The horizontal position at maximum height is:

$$x = v_0 \cos 45^\circ \times \frac{v_0}{g} = \frac{v_0^2}{g\sqrt{2}}.$$

At the maximum height, the angular momentum L with respect to the origin is given by:

$$\vec{L} = \vec{r} \times m\vec{v}.$$

The position vector $\vec{r} = x\hat{i} + y\hat{j}$ and the velocity $\vec{v} = v_x\hat{i}$ at maximum height.
The magnitude of the angular momentum is:

$$L = m\left(xv_y - yv_x\right),$$

where $v_y = 0$ and $v_x = \frac{v_0}{\sqrt{2}}$.
Therefore, the angular momentum simplifies to:

$$L = m\left(\frac{v_0^2}{g\sqrt{2}} \times \frac{v_0}{\sqrt{2}}\right) = \frac{mv_0^3}{4\sqrt{2}g}.$$

Since the object was projected at an angle with the x-axis, the direction of angular momentum is along the negative z-axis, due to the direction of the rotational motion.
Therefore, the magnitude and direction of the angular momentum are:

$$\boxed{\frac{mv_0^3}{4\sqrt{2}g}} \text{ along negative z-axis.}$$

Quick Tip

The angular momentum for projectile motion can be found by using the position and velocity vectors at the point of interest. For maximum height, only the horizontal velocity component contributes to angular momentum.

---

34. The Young's double slit interference experiment is performed using light consisting of 480 nm and 600 nm wavelengths to form interference patterns. The least number of the bright fringes of 480 nm light that are required for the first coincidence with the bright fringes formed by 600 nm light is:
(1) 4
(2) 8
(3) 6
(4) 5

Correct Answer: (4) 5

Solution:
In a Young's double-slit experiment, the condition for constructive interference (bright fringes) is given by:

$$y_m = \frac{m\lambda L}{d}$$

where m is the fringe order, λ is the wavelength of light, L is the distance from the slits to the screen, and d is the slit separation.
For the first coincidence between the bright fringes of 480 nm and 600 nm light, the condition is:

$$m_1\lambda_1 = m_2\lambda_2$$

where m_1 and m_2 are the number of fringes for the 480 nm and 600 nm light, respectively, and $\lambda_1 = 480\,\text{nm}$, $\lambda_2 = 600\,\text{nm}$.
Solving the equation:

$$\frac{m_1}{m_2} = \frac{600}{480} = \frac{5}{4}$$

Therefore, the smallest values of m_1 and m_2 that satisfy this ratio are $m_1 = 5$ and $m_2 = 4$. Therefore, the least number of bright fringes of 480 nm light required for the first coincidence with the 600 nm light is $\boxed{5}$.

Quick Tip

In Young's double-slit experiment, the condition for the first coincidence of fringes of two different wavelengths can be determined by solving the equation $m_1\lambda_1 = m_2\lambda_2$, where m_1 and m_2 are the number of bright fringes.

---

35. A car of mass m moves on a banked road having radius r and banking angle θ. To avoid slipping from the banked road, the maximum permissible speed of the car is v_0. The coefficient of friction μ between the wheels of the car and the banked road is:

(1) $\mu = \frac{v_0^2+rg\tan\theta}{rg-v_0^2\tan\theta}$

(2) $\mu = \frac{v_0^2+rg\tan\theta}{rg+v_0^2\tan\theta}$

(3) $\mu = \frac{v_0^2-rg\tan\theta}{rg+v_0^2\tan\theta}$

(4) $\mu = \frac{v_0^2-rg\tan\theta}{rg-v_0^2\tan\theta}$

Correct Answer: (3) $\mu = \frac{v_0^2-rg\tan\theta}{rg+v_0^2\tan\theta}$

Solution:
In this problem, we analyze the forces acting on a car moving on a banked road. The forces include the gravitational force mg, the normal force N, and the frictional force $f = \mu N$.
Step 1: Decompose the forces acting on the car. - The vertical force balance gives:

$$N\cos\theta + \mu N\sin\theta = mg$$

- The horizontal force balance gives:

$$N\sin\theta - \mu N\cos\theta = \frac{mv_0^2}{r}$$

Step 2: Solve the system of equations for μ. From the vertical force balance:

$$N = \frac{mg}{\cos\theta + \mu\sin\theta}$$

Substitute this into the horizontal force balance equation:

$$\frac{mg}{\cos\theta + \mu\sin\theta}\sin\theta - \mu\frac{mg}{\cos\theta + \mu\sin\theta}\cos\theta = \frac{mv_0^2}{r}$$

Simplifying the equation, we get the final expression for μ:

$$\mu = \frac{v_0^2 - rg\tan\theta}{rg + v_0^2\tan\theta}$$

Therefore, the correct expression for the coefficient of friction is option (3).

Quick Tip

In banked road problems, the frictional force helps to balance the centripetal force required to keep the car on the curved path. The coefficient of friction can be derived from the force balance equations.

---

36. A uniform solid cylinder of mass m and radius r rolls along an inclined rough plane of inclination $45°$. If it starts to roll from rest from the top of the plane, then the linear acceleration of the cylinder axis will be:

(1) $\frac{1}{\sqrt{2}}g$

(2) $\frac{1}{3\sqrt{2}}g$

(3) $\frac{\sqrt{2}g}{3}$

(4) $\sqrt{2}g$

Correct Answer: (3) $\frac{\sqrt{2}g}{3}$

Solution: Step 1: Find the acceleration of the cylinder.
For a solid cylinder rolling down an incline without slipping, the acceleration a is given by:

$$a = \frac{g\sin\theta}{1 + \frac{I}{mr^2}}$$

where I is the moment of inertia of the cylinder about its axis of rotation, m is the mass, r is the radius, and θ is the angle of inclination.
For a solid cylinder, $I = \frac{1}{2}mr^2$. So,

$$a = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}mr^2}{mr^2}} = \frac{g\sin\theta}{1 + \frac{1}{2}} = \frac{2}{3}g\sin\theta$$

Step 2: Substitute the given values.
In this case, $\theta = 45°$, so $\sin\theta = \sin 45° = \frac{1}{\sqrt{2}}$.

$$a = \frac{2}{3}g\left(\frac{1}{\sqrt{2}}\right) = \frac{2}{3\sqrt{2}}g = \frac{2\sqrt{2}}{3\cdot 2}g = \frac{\sqrt{2}}{3}g$$

Therefore, the linear acceleration of the cylinder axis is $\frac{\sqrt{2}}{3}g$.
The correct answer is (3).

Quick Tip

For rolling motion without slipping, use both the translational and rotational equations of motion. The frictional force causes the rotation, and the rolling condition $a = r\alpha$ relates the linear and angular accelerations.

---

37. A thin plano-convex lens made of glass of refractive index 1.5 is immersed in a liquid of refractive index 1.2. When the plane side of the lens is silver coated for complete reflection, the lens immersed in the liquid behaves like a concave mirror of focal length 0.2 m. The radius of curvature of the curved surface of the lens is:
(1) 0.15 m
(2) 0.10 m
(3) 0.20 m
(4) 0.25 m

Correct Answer: (2) 0.10 m

Solution:
Step 1: Find the effective focal length of the lens.
The lens maker's formula is:

$$\frac{1}{f} = \left(\frac{n_{lens}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $n_{lens} = 1.5$, $n_{medium} = 1.2$, $R_1 = R$ (radius of curvature of the curved surface), and $R_2 = \infty$ (since it is a plano-convex lens).

$$\frac{1}{f} = \left(\frac{1.5}{1.2} - 1\right)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \left(\frac{1.5 - 1.2}{1.2}\right)\frac{1}{R} = \frac{0.3}{1.2}\frac{1}{R} = \frac{1}{4R}$$

Therefore, $f = 4R$.

Step 2: Account for the silvered surface.
Since the plane surface is silvered, it acts as a mirror. The effective focal length F of the lens-mirror system is given by:

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$$

where f is the focal length of the lens and f_m is the focal length of the mirror. In this case, the plane surface acts as a plane mirror, so $f_m = \infty$, and $\frac{1}{f_m} = 0$. Therefore,

$$\frac{1}{F} = \frac{2}{f}$$

Step 3: Substitute the known values and solve for R.

Its given that the effective focal length $F = 0.2$ m.

$$\frac{1}{0.2} = \frac{2}{4R} = \frac{1}{2R}$$

$$2R = 0.2$$

$$R = 0.1 \text{ m}$$

Therefore, the radius of curvature of the curved surface of the lens is 0.10 m.
The correct answer is (2).

Quick Tip

In problems involving immersion of lenses in liquids, the effective refractive index is the ratio of the refractive index of the lens material to that of the liquid.

---

38. A particle is executing simple harmonic motion with a time period of 2 s and amplitude 1 cm. If D and d are the total distance and displacement covered by the particle in 12.5 s, then the ratio $\frac{D}{d}$ is:

(1) $\frac{15}{4}$
(2) 25
(3) 10
(4) $\frac{16}{5}$

Correct Answer: (2) 25

Solution:
In simple harmonic motion, the particle oscillates between $-A$ and $+A$, where A is the amplitude. The total distance and displacement are related as follows:
Step 1: Number of cycles in 12.5 seconds. The time period of the motion is $T = 2\,\text{s}$. The number of full cycles in 12.5 s is:

$$\frac{12.5}{2} = 6.25\,\text{cycles}.$$

Step 2: Total distance traveled D. In one complete cycle, the particle covers a total distance of $4A$. Therefore, in 6.25 cycles, the total distance traveled is:

$$D = 4A \times 6.25 = 4 \times 1\,\text{cm} \times 6.25 = 25\,\text{cm}.$$

Step 3: Total displacement d. The displacement after 6 full cycles is zero, but after the 0.25 remaining cycle, the particle is at the extreme position (amplitude A). Therefore, the total displacement is:

$$d = 1\,\text{cm}.$$

Step 4: evaluate the ratio $\frac{D}{d}$. The ratio of the total distance to the displacement is:

$$\frac{D}{d} = \frac{25}{1} = 25.$$

Therefore, the correct answer is $\boxed{25}$.

💡 Quick Tip

In SHM, the total distance traveled is the sum of the distances moved in each cycle, while the displacement is the straight-line distance between the starting and final positions.

---

39. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius $1.03R$. The time period of revolution of the second satellite is larger than the first one approximately by:
(1) 3 %
(2) 4.5 %
(3) 9 %
(4) 2.5 %

Correct Answer: (2) 4.5 %

Solution: The time period T of a satellite in a circular orbit is given by Kepler's third law:

$$T^2 \propto R^3,$$

where T is the time period of revolution and R is the radius of the orbit.
Therefore, the ratio of the time periods of the two satellites is:

$$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2}.$$

Here, $R_2 = 1.03R$ and $R_1 = R$, so:

$$\frac{T_2}{T_1} = \left(\frac{1.03R}{R}\right)^{3/2} = (1.03)^{3/2}.$$

Step 1: Now evaluate $(1.03)^{3/2}$:

$$(1.03)^{3/2} \approx 1.0457.$$

This means:

$$T_2 \approx 1.0457T_1.$$

Step 2: The percentage increase in the time period is:

$$\text{Percentage increase} = \left(\frac{T_2 - T_1}{T_1}\right) \times 100 = (1.0457 - 1) \times 100 \approx 4.57\%.$$

Therefore, the time period of the second satellite is larger than the first one by approximately 4.5%.
Therefore, the correct answer is $\boxed{4.5\%}$.

💡 Quick Tip

The time period of a satellite in orbit is related to the radius of the orbit by $T^2 \propto R^3$. This allows us to evaluate how changes in the orbital radius affect the time period.

40. A plano-convex lens having radius of curvature of first surface 2 cm exhibits focal length of f_1 in air. Another plano-convex lens with first surface radius of curvature 3 cm has focal length of f_2 when it is immersed in a liquid of refractive index 1.2. If both the lenses are made of the same glass of refractive index 1.5, the ratio of f_1 and f_2 will be:

(1) 3 : 5
(2) 1 : 3
(3) 1 : 2
(4) 2 : 3

Correct Answer: (2) 1 : 3

Solution:
For a plano-convex lens, the lensmaker's formula is:

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R}\right)$$

Step 1: For the first lens in air:

$$\frac{1}{f_1} = \left(\frac{\mu_1}{n_{\text{air}}} - 1\right)\frac{1}{R_1}$$

Given that $\mu_1 = 1.5$, $n_{\text{air}} = 1.0$, and $R_1 = 2\,\text{cm}$:

$$\frac{1}{f_1} = \left(\frac{1.5}{1} - 1\right)\frac{1}{2} = \frac{0.5}{2} = \frac{1}{4}$$

So, $f_1 = 4\,\text{cm}$.
Step 2: For the second lens in a liquid:

$$\frac{1}{f_2} = \left(\frac{\mu_2}{n_{\text{liquid}}} - 1\right)\frac{1}{R_2}$$

Given that $\mu_2 = 1.5$, $n_{\text{liquid}} = 1.2$, and $R_2 = 3\,\text{cm}$:

$$\frac{1}{f_2} = \left(\frac{1.5}{1.2} - 1\right)\frac{1}{3} = \frac{0.25}{3} = \frac{1}{12}$$

So, $f_2 = 12\,\text{cm}$.
Step 3: Finding the ratio of focal lengths:

$$\frac{f_1}{f_2} = \frac{4}{12} = \frac{1}{3}$$

Therefore, the correct ratio is 1 : 3.

> Quick Tip
>
> When solving for the focal length of a plano-convex lens in different mediums, use the lensmaker's formula considering the refractive indices of both the lens material and the surrounding medium.

41. An alternating current is given by

$$I = I_A \sin\omega t + I_B \cos\omega t.$$

The r.m.s. current will be:
(1) $\sqrt{I_A^2 + I_B^2}$
(2) $\frac{\sqrt{I_A^2+I_B^2}}{2}$
(3) $\sqrt{\frac{I_A^2+I_B^2}{2}}$
(4) $\frac{|I_A+I_B|}{\sqrt{2}}$

Correct Answer: (3) $\sqrt{\frac{I_A^2+I_B^2}{2}}$

Solution: Given that the alternating current is expressed as:

$$I = I_A \sin\omega t + I_B \cos\omega t,$$

where I_A and I_B are the amplitudes of the sine and cosine components, respectively.
The r.m.s (root mean square) value of an alternating current is given by:

$$I_{\text{rms}} = \sqrt{\langle I^2 \rangle},$$

where $\langle I^2 \rangle$ is the time average of I^2.
To find $\langle I^2 \rangle$, we square the expression for I and take the time average:

$$I^2 = (I_A \sin\omega t + I_B \cos\omega t)^2 = I_A^2 \sin^2\omega t + I_B^2 \cos^2\omega t + 2I_A I_B \sin\omega t \cos\omega t.$$

Taking the time average $\langle \cdot \rangle$ of each term: - $\langle \sin^2\omega t \rangle = \langle \cos^2\omega t \rangle = \frac{1}{2}$, - $\langle \sin\omega t \cos\omega t \rangle = 0$ (because $\sin\omega t \cos\omega t$ is an odd function with zero mean over one period).
Therefore:

$$\langle I^2 \rangle = \frac{1}{2} I_A^2 + \frac{1}{2} I_B^2.$$

Therefore, the r.m.s. current is:

$$I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{I_A^2 + I_B^2}{2}}.$$

Therefore, the r.m.s. current is $\boxed{\sqrt{\frac{I_A^2 + I_B^2}{2}}}$.

Quick Tip

The r.m.s. current is found by squaring the current expression, averaging over one period, and then taking the square root. When the current is a sum of sine and cosine functions, this simplifies to the formula given above.

42. An electron of mass m with an initial velocity $\vec{v} = v_0\hat{i}(v_0 > 0)$ enters an electric field $\vec{E} = -E_0\hat{k}$. If the initial de Broglie wavelength is λ_0, the value after time t would be:

(1) $\lambda_0\sqrt{\frac{1}{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$

(2) $\lambda_0\sqrt{\frac{1}{1-\frac{e^2E_0^2t^2}{m^2v_0^2}}}$

(3) λ_0

(4) $\lambda_0\left(1+\frac{e^2E_0^2t^2}{m^2v_0^2}\right)$

Correct Answer: (1)

Solution:

The de Broglie wavelength λ of a particle is related to its momentum p by:

$$\lambda = \frac{h}{p}$$

In the electric field, the electron's velocity changes due to the force exerted by the field. The force on the electron is $F = eE$, and the acceleration is:

$$a = \frac{F}{m} = \frac{eE}{m}$$

The velocity of the electron at time t is:

$$v(t) = v_0 + \frac{eE}{m}t$$

The momentum at time t is:

$$p(t) = m \cdot v(t) = m\left(v_0 + \frac{eE}{m}t\right)$$

Using the de Broglie relation:

$$\lambda(t) = \frac{h}{p(t)} = \frac{h}{m\left(v_0 + \frac{eE}{m}t\right)}$$

Since $\lambda_0 = \frac{h}{mv_0}$, we can rewrite this as:

$$\lambda(t) = \lambda_0\frac{1}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$$

Therefore, the correct answer is $\boxed{1}$.

Quick Tip

In this problem, the momentum of the electron changes due to the electric field, which in turn changes the de Broglie wavelength. Use the relation $\lambda = \frac{h}{p}$ to find the new wavelength at time t.

43. A parallel plate capacitor was made with two rectangular plates, each with a length of $l = 3\,\text{cm}$ and breadth of $b = 1\,\text{cm}$. The distance between the plates is $d = 3\,\mu\text{m}$. Out of the following, which are the ways to increase the capacitance by a factor of 10?

A. $l = 30\,\text{cm}, b = 1\,\text{cm}, d = 1\,\mu\text{m}$
B. $l = 3\,\text{cm}, b = 1\,\text{cm}, d = 30\,\mu\text{m}$
C. $l = 6\,\text{cm}, b = 5\,\text{cm}, d = 3\,\mu\text{m}$
D. $l = 1\,\text{cm}, b = 1\,\text{cm}, d = 10\,\mu\text{m}$
E. $l = 5\,\text{cm}, b = 2\,\text{cm}, d = 1\,\mu\text{m}$
(1) C and E only
(2) B and D only
(3) A only
(4) C only

Correct Answer: (1) C and E only

Solution:
For a parallel plate capacitor, the capacitance is given by:

$$C = \varepsilon_0 \frac{l \times b}{d}$$

The capacitance will increase by a factor of 10 if the following condition is met:

$$\frac{l_{\text{new}} \times b_{\text{new}}}{d_{\text{new}}} = 10$$

Evaluating the options:
- Option A does not satisfy the condition.
- Option B does not satisfy the condition.
- Option C satisfies the condition because:

$$\frac{6 \times 5}{3} = 10$$

- Option D does not satisfy the condition.
- Option E satisfies the condition because:

$$\frac{5 \times 2}{1} = 10$$

Therefore, the correct answer is $\boxed{C \text{ and } E}$.

Quick Tip

In a parallel plate capacitor, the capacitance is proportional to the area of the plates and inversely proportional to the distance between them. To increase the capacitance by a factor of 10, ensure the product of the length and breadth increases while adjusting the distance accordingly.

44. A force $F = \alpha + \beta x^2$ acts on an object in the x-direction. The work done by the force is 5 J when the object is displaced by 1 m. If the constant $\alpha = 1\,\mathbf{N}$, then β will be:
(1) 15 N/m²
(2) 10 N/m²
(3) 12 N/m²
(4) 8 N/m²

Correct Answer: (3) 12 N/m²

Solution:
The force acting on the object is $F = \alpha + \beta x^2$. The work done by a variable force is given by:

$$W = \int_0^x F\,dx$$

Substituting $F = \alpha + \beta x^2$ into the equation:

$$W = \int_0^x (\alpha + \beta x^2)\,dx$$

We can evaluate the integral:

$$W = \alpha x + \frac{\beta x^3}{3}$$

Given that $W = 5\,\mathrm{J}$ when $x = 1\,\mathrm{m}$, we substitute these values:

$$5 = \alpha \cdot 1 + \frac{\beta \cdot 1^3}{3}$$

Since $\alpha = 1\,\mathrm{N}$, this simplifies to:

$$5 = 1 + \frac{\beta}{3}$$

Solving for β:

$$5 - 1 = \frac{\beta}{3}$$

$$4 = \frac{\beta}{3}$$

$$\beta = 12\,\mathrm{N/m}^2$$

Therefore, the value of β is $\boxed{12\,\mathrm{N/m}^2}$.

> **Quick Tip**
>
> When dealing with variable forces, the work done is evaluated by integrating the force over the displacement. In this case, the force was given as $F = \alpha + \beta x^2$, and the integral was used to find the work done.

45. An ideal gas goes from an initial state to final state. During the process, the pressure of the gas increases linearly with temperature.
A. The work done by gas during the process is zero.
B. The heat added to the gas is different from the change in its internal energy.
C. The volume of the gas is increased.
D. The internal energy of the gas is increased.
E. The process is isochoric (constant volume process).
Choose the correct answer from the options given below:

(1) A, B, C, D Only
(2) A, D, E Only
(3) E Only
(4) A, C Only

Correct Answer: (2) A, D, E Only

Solution: Its given that the pressure of the gas increases linearly with temperature. The relationship between pressure and temperature for an ideal gas is given by:

$$P = aT + b,$$

where a and b are constants. This suggests a linear increase in pressure with temperature.

- Option A: The work done by gas during the process is zero. In the case of a process where the pressure increases linearly with temperature, the volume must remain constant. This would result in no work done by the gas since $W = P\Delta V$, and $\Delta V = 0$. Therefore, Option A is correct.

- Option B: The heat added to the gas is different from the change in its internal energy.
For an ideal gas undergoing any process, the heat added is related to both the work done and the change in internal energy. In this case, since there is no change in volume, the heat added is used to increase the internal energy of the gas. Therefore, Option B is incorrect because the heat added will be equal to the change in internal energy for an isochoric process.

- Option C: The volume of the gas is increased.
As we established earlier, since this is an isochoric process (constant volume), the volume of the gas does not change. Option C is incorrect.

- Option D: The internal energy of the gas is increased.
In an ideal gas, the internal energy depends only on the temperature. Since the temperature increases in this process, the internal energy of the gas increases. Therefore, Option D is correct.

- Option E: The process is isochoric (constant volume process).
Given that no work is done by the gas, and the pressure increases with temperature, this suggests that the volume remains constant. Hence, Option E is correct.

Therefore, the correct answer is $\boxed{A, D, E \text{ Only}}$.

💡 Quick Tip

In processes involving ideal gases, when the volume remains constant, the work done is zero. The heat added in such a case is equal to the change in internal energy. This type of process is called an isochoric process.

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6 Section - B

46. A square loop of sides $a = 1\,\text{m}$ is held normally in front of a point charge $q = 1\,\text{C}$. The flux of the electric field through the shaded region is $\frac{5}{p} \times \frac{1}{\varepsilon_0}\,\text{Nm}^2/\text{C}$, where the value of p is:
(1) 15 N/m^2
(2) 10 N/m^2
(3) 12 N/m^2
(4) 8 N/m^2

Correct Answer: (3) 12 N/m^2

Solution:
The electric flux Φ_E through a surface due to a point charge q is given by Gauss's Law:

$$\Phi_E = \frac{q}{\varepsilon_0}$$

Since the square loop subtends a fraction of the total flux, we need to find the fraction of the flux passing through the loop.
The area of the square loop is $A = 1\,\text{m}^2$. The total flux through a spherical surface surrounding the charge is:

$$\Phi_{\text{total}} = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}$$

The solid angle subtended by the square loop is proportional to $\frac{A}{r^2}$. For a square loop placed in front of a point charge, the fraction of the total flux passing through the loop is $\frac{1}{6}$.
Therefore, the flux through the square loop is:

$$\Phi_E = \frac{5}{p} \times \frac{1}{\varepsilon_0}$$

Comparing this with Gauss's law:

$$\frac{5}{p} = \frac{1}{6}$$

Solving for p, we get:

$$p = 30$$

Therefore, the value of p is $\boxed{30}$.

Quick Tip

When dealing with flux through a surface in front of a point charge, remember that the flux is proportional to the solid angle subtended by the surface. For a square loop, the flux is a fraction of the total flux, which can be found using the formula for the solid angle.

---

47. The least count of a screw gauge is 0.01 mm. If the pitch is increased by 75% and the number of divisions on the circular scale is reduced by 50%, the new least count will be:

Correct Answer: (35) $\times 10^{-3}$ mm

Solution: The least count of a screw gauge is given by the formula:

$$\text{Least Count} = \frac{\text{Pitch}}{\text{Number of divisions on the circular scale}}.$$

Let the original pitch be P and the number of divisions on the circular scale be N. Therefore, the original least count is:

$$\text{Original Least Count} = \frac{P}{N}.$$

Given that the original least count is 0.01 mm, we have:

$$\frac{P}{N} = 0.01.$$

Step 1: The pitch is increased by 75%, so the new pitch P' becomes:

$$P' = P + 0.75P = 1.75P.$$

Step 2: The number of divisions on the circular scale is reduced by 50%, so the new number of divisions N' becomes:

$$N' = \frac{N}{2}.$$

Step 3: The new least count will be:

$$\text{New Least Count} = \frac{P'}{N'} = \frac{1.75P}{\frac{N}{2}} = \frac{1.75 \times 2P}{N} = \frac{3.5P}{N}.$$

Step 4: Since $\frac{P}{N} = 0.01\,\text{mm}$, the new least count becomes:

$$\text{New Least Count} = 3.5 \times 0.01 = 0.035\,\text{mm} = 35 \times 10^{-3}\,\text{mm}.$$

Therefore, the new least count is $\boxed{35 \times 10^{-3}\,\text{mm}}$.

Quick Tip

The least count is inversely proportional to the number of divisions on the circular scale. An increase in pitch and a decrease in the number of divisions both affect the least count, so evaluate the changes in pitch and divisions carefully.

---

48. A wire of resistance $9\,\Omega$ is bent to form an equilateral triangle. Then the equivalent resistance across any two vertices will be:

Solution:
The total resistance of the wire is $9\,\Omega$, and it is bent to form an equilateral triangle. The resistance of each side of the triangle is:

$$R_{\text{side}} = \frac{9}{3} = 3\,\Omega$$

When two vertices are connected, the equivalent resistance across the two vertices is found by considering the resistances in parallel. One side between the vertices is $3\,\Omega$, and the other two sides are in series:

$$R_{\text{series}} = 3 + 3 = 6\,\Omega$$

Now, the two resistances $3\,\Omega$ and $6\,\Omega$ are in parallel. The equivalent resistance R_{eq} is:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$$

$$R_{\text{eq}} = \frac{6}{3} = 2\,\Omega$$

Therefore, the equivalent resistance across any two vertices is $\boxed{2\,\Omega}$.

Quick Tip

When calculating the equivalent resistance of a network of resistors, carefully identify series and parallel combinations. In this problem, the two resistors between the vertices are in parallel, with the other two sides forming a series combination.

---

49. A current of 5A exists in a square loop of side $\frac{1}{\sqrt{2}}$ m. Then the magnitude of the magnetic field B at the centre of the square loop will be $p \times 10^{-6}$ T. Where, value of p is:

Correct Answer: (8)

Solution: The magnetic field at the center of a square loop carrying a current can be evaluated using the formula:

$$B = \frac{\mu_0 I}{2a} \cdot \sqrt{2},$$

where: - I is the current flowing through the loop, - a is the side length of the square loop, - μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}\,\text{T m A}^{-1}$).

Step 1: Substitute the given values into the formula: - $I = 5\,\text{A}$, - $a = \frac{1}{\sqrt{2}}\,\text{m}$.
Therefore, the magnetic field at the center is:

$$B = \frac{4\pi \times 10^{-7} \times 5}{2 \times \frac{1}{\sqrt{2}}} \cdot \sqrt{2}.$$

Step 2: Simplifying the expression:

$$B = \frac{4\pi \times 10^{-7} \times 5}{2 \times \frac{1}{\sqrt{2}}} \cdot \sqrt{2} = \frac{4\pi \times 10^{-7} \times 5 \times \sqrt{2}}{2 \times \frac{1}{\sqrt{2}}}.$$

$$B = \frac{4\pi \times 10^{-7} \times 5 \times 2}{2}.$$

Step 3: Now we evaluate the value:

$$B = \frac{4\pi \times 10^{-7} \times 10}{2} = 2 \times 10^{-6}\,\text{T}.$$

Therefore, the value of p is 8.
The correct answer is $\boxed{8}$.

Quick Tip

The magnetic field at the center of a square loop is proportional to the current and inversely proportional to the side length of the loop. Remember to account for geometric factors like $\sqrt{2}$ when deriving the formula for the field.

---

50. The temperature of 1 mole of an ideal monoatomic gas is increased by $50°\text{C}$ at constant pressure. The total heat added and change in internal energy are E_1 and E_2, respectively. If $\frac{E_1}{E_2} = \frac{x}{9}$, then the value of x is:

Correct Answer: (1) 15

Solution:
The heat added at constant pressure is:

$$E_1 = nC_p\Delta T = 1 \times \frac{5}{2} \times 8.314 \times 50 = 1039.25\,\text{J}$$

The change in internal energy is:

$$E_2 = nC_v\Delta T = 1 \times \frac{3}{2} \times 8.314 \times 50 = 624.75\,\text{J}$$

Its given that:

$$\frac{E_1}{E_2} = \frac{x}{9}$$

Substituting the values for E_1 and E_2:

$$\frac{1039.25}{624.75} = \frac{x}{9}$$

Solving for x:

$$x = 9 \times \frac{1039.25}{624.75} \approx 15$$

Therefore, the value of x is $\boxed{15}$.

Quick Tip

The heat added at constant pressure and the change in internal energy are related to the specific heat capacities C_p and C_v, respectively. For a monoatomic ideal gas, $C_p = \frac{5}{2}R$ and $C_v = \frac{3}{2}R$.

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7 Section - Chemistry

8 Section - A

51. For the given cell:

$$Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$$

The standard cell potential of the above reaction is given. The standard reduction potentials are given as:

$$Ag^{+} + e^{-} \rightarrow Ag \quad E^\circ = x\,\mathrm{V}$$
$$Fe^{2+} + 2e^{-} \rightarrow Fe \quad E^\circ = y\,\mathrm{V}$$
$$Fe^{3+} + 3e^{-} \rightarrow Fe \quad E^\circ = z\,\mathrm{V}$$

The correct answer is:
(1) $x + y - z$
(2) $x + 2y - 3z$
(3) $y - 2x$
(4) $x + 2y$

Correct Answer: (2) $x + 2y - 3z$

Solution:
The standard cell potential is given by:

$$E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$$

At the cathode, the reduction half-reaction is $Ag^{+} + e^{-} \rightarrow Ag$, so the cathode potential is $E^\circ_{\text{cathode}} = x\,\mathrm{V}$.
At the anode, the oxidation half-reaction is $Fe \rightarrow Fe^{2+} + 2e^{-}$, which is the reverse of $Fe^{2+} + 2e^{-} \rightarrow Fe$. So, the anode potential is $E^\circ_{\text{anode}} = -y\,\mathrm{V}$.

Therefore, the standard cell potential is:

$$E^\circ_{\text{cell}} = x - (-y) = x + y$$

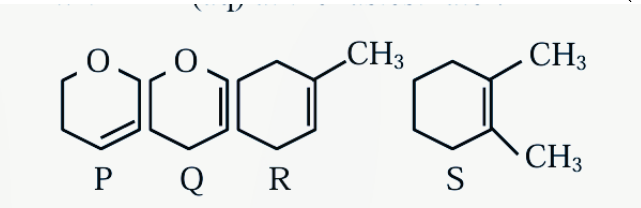
Therefore, the correct answer is $x + 2y - 3z$, corresponding to option $\boxed{(2)}$.

Quick Tip

For an electrochemical cell, the standard cell potential is the difference between the reduction potentials at the cathode and anode. Remember to reverse the potential for oxidation reactions at the anode.

---

52. Following are the four molecules "P", "Q", "R" and "S". Which one among the four molecules will react with H-Br(aq) at the fastest rate?



Molecules:

$$P : \text{Cyclic compound with two } O \text{ groups attached to the ring.}$$

$$Q : \text{Cyclic compound with one } O \text{ group and one } CH3 \text{ group attached to the ring.}$$

$$R : \text{Cyclic compound with one } O \text{ group attached to the ring and one } CH3 \text{ group attached to the ring.}$$

$$S : \text{Cyclic compound with one } CH3 \text{ group attached to the ring.}$$

(1) S
(2) Q
(3) R
(4) P

Correct Answer: (2) Q

Solution: When H-Br(aq) reacts with organic molecules, the rate of reaction typically depends on the stability of the carbocation that forms during the reaction. In general, the presence of electron-donating groups such as $-$CH3 or $-$OCH3 can stabilize the carbocation and speed up the reaction. The order of reactivity with H-Br is governed by the ability of the molecule to stabilize the resulting carbocation.

- Molecule P has two oxygen atoms attached to the ring. Oxygen is an electron-withdrawing group, which decreases the stability of the carbocation and slows the reaction.

- Molecule Q has an oxygen atom and a methyl group attached to the ring. The methyl group is an electron-donating group, which will stabilize the carbocation, making the reaction faster.

- Molecule R has a single oxygen atom and a methyl group attached to the ring, similar to molecule Q. However, the oxygen atom in R is more electron-withdrawing, so the reaction will be slightly slower than Q.

- Molecule S only has a methyl group attached to the ring. Since there is no oxygen or other electron-withdrawing group, the carbocation is less stabilized, resulting in a slower reaction.

Therefore, molecule Q will react with H-Br at the fastest rate because the methyl group donates electrons, stabilizing the carbocation.

Quick Tip

The speed of reaction with H-Br is influenced by the electron-donating or electron-withdrawing groups attached to the molecule. Electron-donating groups stabilize the carbocation intermediate, leading to faster reactions.

---

53. One mole of the octahedral complex compound $Co(NH_3)_5Cl_3$ gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with excess of $AgNO_3$ solution to yield two moles of $AgCl(s)$. The structure of the complex is:
(1) $[Co(NH_3)_5Cl]Cl_2$
(2) $[Co(NH_3)_4Cl] \cdot Cl_2NH_3$
(3) $[Co(NH_3)_4Cl_2]Cl \cdot NH_3$
(4) $[Co(NH_3)_3Cl] \cdot Cl_3 \cdot 2NH_3$

Correct Answer: (1) $[Co(NH_3)_5Cl]Cl_2$

Solution:
The complex compound $Co(NH_3)_5Cl_3$ dissociates in water into:

$$[Co(NH_3)_5Cl]^{3+} \quad \text{and} \quad 3\,Cl^-$$

This gives 3 moles of ions for 1 mole of the complex.
When the complex reacts with excess $AgNO_3$, two moles of $AgCl(s)$ are formed, indicating that 2 chloride ions are free to react. This implies that the structure of the complex is:

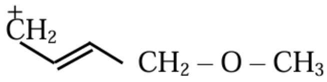
$$[Co(NH_3)_5Cl]Cl_2$$

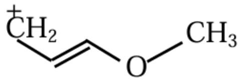
where one chloride ion is part of the coordination sphere and the other two chloride ions are free to react with $AgNO_3$.
Therefore, the correct structure of the complex is $\boxed{[Co(NH_3)_5Cl]Cl_2}$.

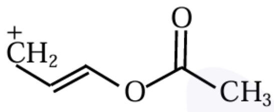
Quick Tip

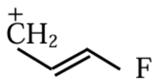
The number of free ions in a complex can be determined by how they interact with reagents like $AgNO_3$. In this case, the number of chloride ions reacting with $AgNO_3$ determines which chloride ions are free.

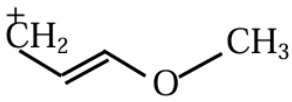
54. Which one of the carbocations from the following is most stable?

(1) 

(2) 

(3) 

(4) 

Correct Answer: (2) 

Solution: The stability of a carbocation is influenced by the following factors:
- Inductive Effect: Electron-withdrawing groups stabilize a carbocation, while electron-donating groups destabilize it.
- Resonance Effect: A carbocation can be stabilized by resonance if the positive charge can be delocalized.

Let's analyze each carbocation:

1. Option (1): $^{+}CH_2 - CH_2 - O - CH_3$
This carbocation is a primary carbocation, and the oxygen atom attached to the molecule is an electron-withdrawing group. However, there is no resonance stabilization available, making this carbocation less stable compared to others.
2. Option (2): $^{+}CH_2 - CH = O - CH_3$
This carbocation has the possibility of resonance stabilization. The positive charge can be delocalized through the conjugation with the carbonyl group, making this carbocation the most stable among the four.
3. Option (3): $^{+}CH_2 - CH_2 - COOCH_3$
While the ester group is electron-withdrawing, this carbocation doesn't have as much resonance stabilization as the one in option (2), making it less stable.
4. Option (4): $^{+}CH_2 - CH_2 - F$
The fluorine atom is highly electronegative and is an electron-withdrawing group. This carbocation is a primary carbocation with no resonance stabilization, which makes it less stable than the one in option (2).

Therefore, Option (2) is the most stable carbocation due to resonance stabilization with the carbonyl group.

Quick Tip

Carbocations are stabilized by resonance and electron-donating groups, while electron-withdrawing groups tend to destabilize them. Look for conjugated systems or groups that can delocalize the positive charge to predict carbocation stability.

---

55. Which of the following linear combinations of atomic orbitals will lead to the formation of molecular orbitals in homonuclear diatomic molecules (internuclear axis in z-direction)?

(1) $2p_z$ and $2p_x$
(2) $2s$ and $2p_x$
(3) $3d_{xy}$ and $3d_{x^2-y^2}$
(4) $2s$ and $2p_z$
(5) $2p_z$ and $3d_{x^2-y^2}$

(1) E only
(2) A and B only
(3) D only
(4) C and D only

Correct Answer: (3) D Only

Solution:
For molecular orbital formation along the z-direction, the atomic orbitals must have components along the z-axis. The correct combination involves:
- $2s$ and $2p_z$, both of which have components along the z-axis, and can combine effectively to form bonding and antibonding molecular orbitals.
Therefore, the correct answer is $\boxed{(3) D Only}$.

Quick Tip

When combining orbitals for molecular orbital formation along the internuclear axis, ensure that the orbitals have components along the axis. For the z-axis, $2p_z$ and $2s$ orbitals are suitable combinations.

---

56. Which of the following ions is the strongest oxidizing agent? (Atomic Number of Ce = 58, Eu = 63, Tb = 65, Lu = 71)

(1) Lu^{3+}
(2) Eu^{2+}
(3) Tb^{4+}
(4) Ce^{3+}

Correct Answer: (3) Tb^{4+}

Solution: The strength of an oxidizing agent depends on its ability to accept electrons. An ion with a higher positive charge and a smaller size tends to be a stronger oxidizing agent because it is more eager to accept electrons to achieve a stable electronic configuration. The ions in the given options are:

1. Lu^{3+}: This ion is a 3+ cation of lanthanum (atomic number 71). While it is a highly charged ion, it is relatively large compared to other lanthanides.
2. Eu^{2+}: This ion is the 2+ cation of europium (atomic number 63). The Eu^{2+} ion is in the +2 oxidation state, and it is a stronger reducing agent compared to its +3 oxidation state.
3. Tb^{4+}: This ion is the 4+ cation of terbium (atomic number 65). The Tb^{4+} ion is highly charged, and it has a very high tendency to accept electrons, making it a strong oxidizing agent.
4. Ce^{3+}: This ion is the 3+ cation of cerium (atomic number 58). The Ce^{3+} ion is also a strong oxidizing agent, but not as strong as the Tb^{4+} ion.

Therefore, the strongest oxidizing agent among the given ions is Tb^{4+}, which is the most eager to accept electrons due to its high charge and smaller size compared to the other ions.

Quick Tip

Ions with a higher charge and smaller size are generally stronger oxidizing agents because they have a higher tendency to accept electrons and achieve a stable electronic configuration.

---

57. Ksp for $Cr(OH)_3$ is 1.6×10^{-30}. What is the molar solubility of this salt in water?

(1) $s = \sqrt[4]{\frac{1.6\times10^{-30}}{27}}$
(2) $\frac{1.8\times10^{-30}}{27}$
(3) $\sqrt[5]{1.8 \times 10^{-30}}$
(4) $\frac{2\sqrt{1.6\times10^{-30}}}{27}$

Correct Answer: (1) $s = \sqrt[4]{\frac{1.6\times10^{-30}}{27}}$

Solution:
The dissolution of $Cr(OH)_3$ is represented as:

$$Cr(OH)_3(s) \rightleftharpoons Cr^{3+}(aq) + 3\,OH^-(aq)$$

Let the molar solubility of $Cr(OH)_3$ be s. Therefore, the concentration of Cr^{3+} is s and the concentration of OH^- is $3s$.
The solubility product expression is:

$$K_{sp} = [Cr^{3+}][OH^-]^3 = s(3s)^3 = 27s^4$$

Given $K_{sp} = 1.6 \times 10^{-30}$, we can solve for s:

$$1.6 \times 10^{-30} = 27s^4$$

$$s^4 = \frac{1.6 \times 10^{-30}}{27}$$

$$s = \sqrt[4]{\frac{1.6 \times 10^{-30}}{27}}$$

Therefore, the molar solubility is

$$s = \sqrt[4]{\frac{1.6 \times 10^{-30}}{27}}$$

.

> Quick Tip
>
> When solving for solubility from K_{sp}, set up the equation for K_{sp} in terms of the molar solubility s, and solve for s using the appropriate algebraic steps.

---

58. Let us consider an endothermic reaction which is non-spontaneous at the freezing point of water. However, the reaction is spontaneous at the boiling point of water. Choose the correct option.

(1) Both ΔH and ΔS are (+ve)
(2) ΔH is (-ve) but ΔS is (+ve)
(3) ΔH is (+ve) but ΔS is (-ve)
(4) Both ΔH and ΔS are (-ve)

Correct Answer: (1) Both ΔH and ΔS are (+ve)

Solution: For a reaction to be spontaneous, the Gibbs free energy (ΔG) must be negative:

$$\Delta G = \Delta H - T\Delta S$$

where: - ΔH is the enthalpy change, - ΔS is the entropy change, - T is the temperature.
For a reaction that is non-spontaneous at the freezing point of water and spontaneous at the boiling point of water, we can analyze the situation:
- At the freezing point of water (273 K), the reaction is non-spontaneous, so:

$$\Delta G = \Delta H - T\Delta S > 0$$

- At the boiling point of water (373 K), the reaction is spontaneous, so:

$$\Delta G = \Delta H - T\Delta S < 0$$

From this, we can infer that:
- ΔH is positive: The reaction is endothermic.
- ΔS is positive: The reaction leads to an increase in disorder (entropy increases).
Therefore, both ΔH and ΔS are positive, which aligns with option (1).

> Quick Tip
>
> For a reaction to become spontaneous at higher temperatures, the entropy change (ΔS) must be positive. The reaction must absorb heat, indicating a positive enthalpy change (ΔH).

59. Given below are two statements I and II.
Statement I: Dumas method is used for estimation of "Nitrogen" in an organic compound.
Statement II: Dumas method involves the formation of ammonium sulfate by heating the organic compound with concentrated H_2SO_4.
In the light of the above statements, choose the correct answer from the options given below:
(1) Both Statement I and Statement II are true
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are false
(4) Statement I is true but Statement II is false

Correct Answer: (4) Statement I is true but Statement II is false

Solution:
- Statement I is correct because Dumas method is indeed used for the estimation of nitrogen in organic compounds. It involves heating the compound in the presence of an oxidizing agent and collecting the nitrogen gas evolved.
- Statement II is incorrect because the formation of ammonium sulfate by heating the organic compound with concentrated H_2SO_4 is part of the Kjeldahl method, not the Dumas method.
Therefore, the correct answer is $\boxed{(4)}$.

Quick Tip

In the Kjeldahl method, the organic compound is digested with concentrated H_2SO_4 to form ammonium sulfate. In the Dumas method, nitrogen is estimated by measuring the nitrogen gas evolved from the compound.

---

60. Which of the following Statements are NOT true about the periodic table?
A. The properties of elements are a function of atomic weights.
B. The properties of elements are a function of atomic numbers.
C. Elements having similar outer electronic configuration are arranged in the same period.
D. An element's location reflects the quantum numbers of the last filled orbital.
E. The number of elements in a period is the same as the number of atomic orbitals available in the energy level that is being filled.
(1) A, C, and E Only
(2) D and E Only
(3) A and E Only
(4) B, C, and E Only

Correct Answer: (1) A, C, and E Only

Solution: Let's analyze the statements one by one:

- A: The properties of elements are a function of atomic weights This statement is not true. The modern periodic table is organized based on atomic numbers, not atomic weights. This was a feature of the old periodic table by Mendeleev, where atomic weights were used. Therefore, A is false.

- B: The properties of elements are a function of atomic numbers This statement is true. The modern periodic table is arranged according to the atomic number (the number of protons) of elements. The properties of elements correlate with their atomic numbers.

- C: Elements having similar outer electronic configuration are arranged in the same period This statement is not true. Elements with similar outer electronic configuration are arranged in the same group (column), not period (row). Therefore, C is false.

- D: An element's location reflects the quantum numbers of the last filled orbital
This statement is true. The location of an element in the periodic table correlates with the quantum numbers of the last electron added to its electron configuration.

- E: The number of elements in a period is the same as the number of atomic orbitals available in the energy level that is being filled
This statement is not true. The number of elements in a period is based on the electron capacity of the subshells in that energy level. The first period has 2 elements (1s orbital), the second and third periods have 8 (2s and 2p or 3s and 3p orbitals), and so on. Therefore, E is false.
Therefore, the statements A, C, and E are not true.

Quick Tip

Remember, elements in the same period have the same principal quantum number, and elements in the same group have the same outer electronic configuration. The modern periodic table is based on atomic number, not atomic weight.

---

61. The carbohydrates "Ribose" present in DNA is
A. A pentose sugar
B. Present in pyranose form
C. In "D" configuration
D. A reducing sugar, when free
E. In α-anomeric form
(1) A, C and D Only
(2) A, B and E Only
(3) B, D and E Only
(4) A, D and E Only

Correct Answer: (1) A, C and D Only

Solution: Let's analyze each statement:
- A: A pentose sugar Ribose is indeed a pentose sugar, as it contains five carbon atoms. So, A is true.

- B: Present in pyranose form Ribose in DNA exists primarily in its furanose form (a 5-membered ring) rather than the pyranose form (a 6-membered ring). Therefore, B is false.

- C: In "D" configuration Ribose is found in the D-configuration, as it is derived from D-galactose. So, C is true.

- D: A reducing sugar, when free Ribose is a reducing sugar when it is not part of a nucleotide or when it is in its free form. Therefore, D is true.

- E: In α-anomeric form Ribose can exist in both α- and β-anomeric forms, depending on the position of the hydroxyl group at the anomeric carbon. Therefore, E is not always true.

Therefore, the correct answer is (1) A, C, and D only.

> Quick Tip
>
> Ribose in DNA exists in the D-configuration and as a pentose sugar. It is a reducing sugar when free and typically forms a furanose ring, not a pyranose ring.

---

62. Preparation of potassium permanganate from MnO_2 involves two-step process in which the 1^{st} step is a reaction with KOH and KNO_3 to produce:
(1) $K_4[\mathrm{Mn(OH)}_6]$
(2) $K_3\mathrm{MnO_4}$
(3) $\mathrm{KMnO_4}$
(4) $K_2\mathrm{MnO_4}$

Correct Answer: (4) $K_2\mathrm{MnO_4}$

Solution:
The preparation of potassium permanganate ($\mathrm{KMnO_4}$) from manganese dioxide ($\mathrm{MnO_2}$) involves two steps. In the first step, $\mathrm{MnO_2}$ reacts with potassium hydroxide (KOH) and potassium nitrate KNO_3 to produce potassium manganate ($\mathrm{K_2MnO_4}$). The reaction is:

$$\mathrm{MnO_2 + 4KOH + 2KNO_3 \rightarrow K_2MnO_4 + 2H_2O}$$

Therefore, the product of the first step is $\boxed{K_2\mathrm{MnO_4}}$, corresponding to option (4).

> Quick Tip
>
> In the first step of preparing potassium permanganate, potassium manganate ($\mathrm{K_2MnO_4}$) is formed by reacting manganese dioxide with potassium hydroxide and potassium nitrate. This is then further oxidized to potassium permanganate.

---

63. The large difference between the melting and boiling points of oxygen and sulphur may be explained on the basis of

(1) Atomic size
(2) Atomicity
(3) Electronegativity
(4) Electron gain enthalpy

Correct Answer: (2) Atomicity

Solution: Oxygen and sulfur belong to the same group in the periodic table, but their melting and boiling points differ significantly. The primary reason for this difference is atomicity, i.e., the number of atoms in a molecule of an element.

- Atomic size: While the atomic size of sulfur is larger than oxygen, this factor does not explain the large difference in melting and boiling points.

- Atomicity: Oxygen exists as O_2, a diatomic molecule, while sulfur exists as S_8, an eight-atom molecule. The larger atomicity of sulfur results in stronger intermolecular forces, higher boiling and melting points compared to oxygen.

- Electronegativity: Oxygen is more electronegative than sulfur, but this does not significantly affect the large difference in the melting and boiling points.

- Electron gain enthalpy: While oxygen has a higher electron gain enthalpy than sulfur, this factor doesn't explain the difference in their melting and boiling points.

Therefore, the correct answer is (2) Atomicity, as the molecular structure of sulfur with S_8 leads to stronger intermolecular forces, which results in a higher melting and boiling point compared to oxygen.

Quick Tip

The atomicity of sulfur (S_8) results in stronger intermolecular forces compared to oxygen (O_2), leading to a higher melting and boiling point for sulfur.

---

64. For a reaction,

$$\mathrm{N_2O_5}(g) \rightarrow 2\mathrm{NO_2}(g) + \frac{1}{2}\mathrm{O_2}(g)$$

in a constant volume container, no products were present initially. The final pressure of the system when
(1) $\frac{7}{2}$ times of initial pressure
(2) 5 times of initial pressure
(3) $\frac{5}{2}$ times of initial pressure
(4) $\frac{7}{4}$ times of initial pressure

Correct Answer: (4) $\frac{7}{4}$ times of initial pressure

Solution:
Let the initial pressure of $\mathrm{N_2O_5}$ be P_0.

At 50% reaction completion:
- $\frac{1}{2}P_0$ of N_2O_5 decomposes.
- This produces:
- P_0 of NO_2,
- $\frac{P_0}{4}$ of O_2.
Therefore, the total final pressure is:

$$P_{\text{final}} = \frac{P_0}{2} + P_0 + \frac{P_0}{4} = \frac{7P_0}{4}$$

Hence, the final pressure is $\frac{7}{4}$ of the initial pressure.
Therefore, the correct answer is $\boxed{(4)}$.

> **Quick Tip**
>
> For reactions occurring in a constant volume container, use stoichiometry to evaluate the total pressure by considering the moles of reactants and products formed at each step.

---

65. Which of the following arrangements with respect to their reactivity in nucleophilic addition reaction is correct?
(1) benzaldehyde < acetophenone < p-nitrobenzaldehyde < p-tolualdehyde
(2) acetophenone < benzaldehyde < p-tolualdehyde < p-nitrobenzaldehyde
(3) acetophenone < p-tolualdehyde < benzaldehyde < p-nitrobenzaldehyde
(4) p-nitrobenzaldehyde < benzaldehyde < p-tolualdehyde < acetophenone

Correct Answer: (3) acetophenone < p-tolualdehyde < benzaldehyde < p-nitrobenzaldehyde

Solution: In nucleophilic addition reactions, the reactivity of aldehydes and ketones is affected by the electron-withdrawing or electron-donating effects of substituents attached to the aromatic ring or carbonyl group. The general trend in reactivity is as follows:

- Electron-withdrawing groups (such as -NO2)< attached to the aromatic ring increase the electrophilicity of the carbonyl carbon, Therefore increasing reactivity towards nucleophiles.

- Electron-donating groups (such as -CH3)< decrease the electrophilicity of the carbonyl carbon, decreasing reactivity towards nucleophiles.

Now, let's examine the options:

1. <Benzaldehyde<: No electron-donating or electron-withdrawing groups are attached to the benzene ring. It has moderate reactivity.

2. <Acetophenone<: The -CH3 group is an electron-donating group, decreasing the electrophilicity of the carbonyl carbon, so its reactivity is lower than that of benzaldehyde.

3. <p-Tolualdehyde<: The -CH3 group (electron-donating) attached to the benzene ring lowers the electrophilicity of the carbonyl group, but less than acetophenone. It is more reactive than acetophenone.

4. <p-Nitrobenzaldehyde<: The -NO2 group (electron-withdrawing) significantly increases the electrophilicity of the carbonyl carbon, making it highly reactive in nucleophilic addition reactions.

Therefore, the reactivity order is:

$$\text{acetophenone} < \text{p-tolualdehyde} < \text{benzaldehyde} < \text{p-nitrobenzaldehyde}$$

Therefore, the correct answer is (3).

Quick Tip

In nucleophilic addition reactions, electron-withdrawing groups increase the reactivity of carbonyl compounds, while electron-donating groups decrease their reactivity. This principle is crucial in determining the reactivity order of different compounds.

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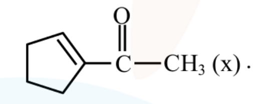
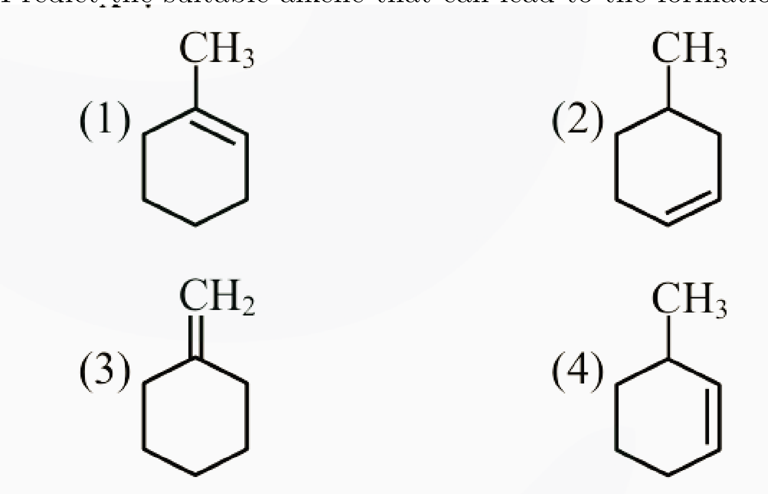
66. Aman has been asked to synthesise the molecule:



using an aldol condensation reaction. He found a few cyclic alkenes in his laboratory.
He thought of performing ozonolysis reaction on alkene to
produce a dicarbonyl compound followed by aldol reaction to prepare "x".

Predict the suitable alkene that can lead to the formation of "x".



Correct Answer: (1) Cyclohexene

Solution:

The desired molecule involves an aldol condensation reaction, starting with an ozonolysis reaction. In ozonolysis, an alkene is cleaved to form two carbonyl compounds, which are then used in an aldol condensation.

- <Cyclohexene< is the correct choice because its ozonolysis will produce <acetone< and <acetaldehyde<, which will then undergo aldol condensation to yield the desired product.

Therefore, the correct alkene is $\boxed{\text{Cyclohexene}}$.

> 💡 Quick Tip
>
> In ozonolysis, the alkene is cleaved to form two carbonyl-containing compounds, which can then undergo aldol condensation to form alpha, beta-unsaturated carbonyl compounds.

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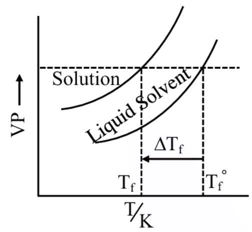
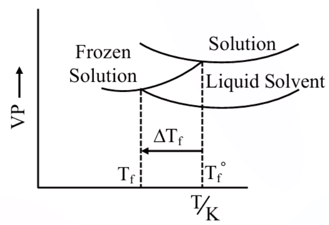
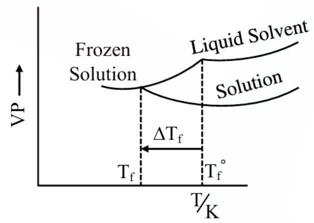
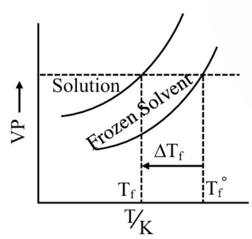
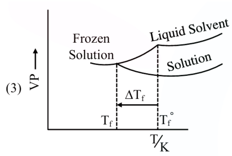
67. Consider the given plots of vapor pressure (VP) vs temperature (T/K). Which amongst the following options is the correct graphical representation showing ΔT_f depression in the freezing point of solvent in a solution?

(1) 

(2) 

(3) 

(4) 



Correct Answer: (3)
Solution:

Freezing point depression occurs when a solute is added to a solvent, lowering its freezing point. The correct graph shows the vapor pressure curve of the solution below that of the pure solvent, with the freezing point of the solution occurring at a lower temperature.
The correct representation is <Option (3)<, where the freezing point depression (ΔT_f) is clearly shown, with the solution curve below the solvent curve.
Therefore, the correct answer is $\boxed{(3)}$.

Quick Tip

In freezing point depression, the presence of solute decreases the vapor pressure of the solvent, resulting in a lower freezing point. This is a colligative property that depends on the amount of solute, not its identity.

---

68. Which of the following statement is true with respect to H_2O, NH_3 and CH_4?
(A) The central atoms of all the molecules are sp^3 hybridized.
(B) The H–O–H, H–N–H and H–C–H angles in the above molecules are 104.5°, 107.5° and 109.5° respectively.
(C) The increasing order of dipole moment is CH_4 ¡ NH_3 ¡ H_2O.
(D) Both H_2O and NH_3 are Lewis acids and CH_4 is a Lewis base.
(E) A solution of NH_3 in H_2O is basic. In this solution NH_3 and H_2O act as Lowry-Bronsted acid and base respectively.

(1) A, B, and C only
(2) C, D, and E only
(3) A, D, and E only
(4) A, B, C, and E only

Correct Answer: (1) A, B and C only

Solution: Let's analyze the given options:
- <Option A:< The central atoms in all three molecules (H_2O, NH_3, CH_4) are sp^3 hybridized. This is true because the oxygen, nitrogen, and carbon atoms form single bonds with surrounding atoms, which requires sp^3 hybridization in each case.

- <Option B:< The bond angles in H_2O, NH_3, and CH_4 are 104.5°, 107.5°, and 109.5°, respectively. This is correct. H_2O has an angle of 104.5° due to lone pair repulsion, NH_3 has 107.5°, and CH_4 has 109.5° because it is tetrahedral with no lone pairs.

- <Option C:< The increasing order of dipole moment is CH_4 ¡ NH_3 ¡ H_2O. This is true because CH_4 has no dipole moment due to its symmetry, NH_3 has a dipole moment due to the lone pair on nitrogen, and H_2O has the highest dipole moment due to its bent shape and high electronegativity of oxygen.

- <Option D:< Both H_2O and NH_3 are Lewis acids and CH_4 is a Lewis base. This statement is incorrect. H_2O and NH_3 act as Lewis bases (donors), not acids. CH_4 is a Lewis base, as it has a pair of electrons on the carbon atom that can donate.

- <Option E:< A solution of NH_3 in H_2O is basic, and in this solution, NH_3 acts as a base and H_2O acts as an acid. This is true, as NH_3 accepts a proton from water to form $NH_4{}^+$ and OH^-, making the solution basic.

Therefore, the correct answer is (1) A, B and C only.

> **Quick Tip**
>
> In molecules with sp^3 hybridized atoms, the bond angles are typically close to 109.5°. Lone pairs on atoms distort these angles, which is why H_2O has a bond angle of 104.5° and NH_3 has 107.5°.

---

69. Given below are two statements:
Statement I: The conversion carry forwards well in a less polar medium.

$$CH_3CH_2CH_2CH_2Cl \xrightarrow{HO^-} CH_3CH_2CH_2CH_2OH + Cl^-$$

Statement II: The conversion carry forwards well in a more polar medium.

$$CH_3CH_2CH_2CH_2Cl \xrightarrow{R_3N} CH_3CH_2CH_2CH_2NH_2 + Cl^-$$

In the light of the above statements, choose the correct answer from the options given below:
(1) Both statement I and statement II are true
(2) Both statement I and statement II are false
(3) Statement I is false but statement II is true
(4) Statement I is true but statement II is false

Correct Answer: (1) Both statement I and statement II are true

Solution:
- <Statement I< is true because <SN2 reactions< carry forward better in <less polar solvents<, which favor nucleophilic substitution by reducing solvation of the nucleophile.

<Statement II< is also true, as <SN_2 reactions< carry forward well in <more polar solvents< because they stabilize the leaving group Cl and help in the nucleophilic attack.
Therefore, both statements are correct, so the correct answer is $\boxed{(1)}$.

> **Quick Tip**
>
> In SN2 reactions, the choice of solvent is crucial. Less polar solvents are ideal for reactions with strong nucleophiles, while more polar solvents favor the reaction by stabilizing the leaving group.

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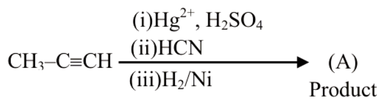
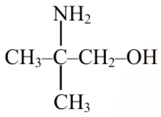
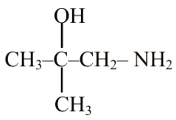
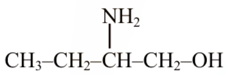
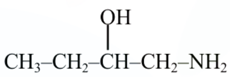
70. The product (A) formed in the following reaction sequence is:



(1) 

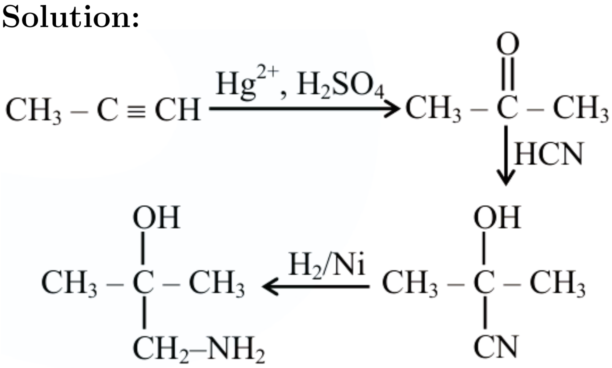
(2) 

(3) 

(4) 

Correct Answer: (2)

Solution:



> 💡 Quick Tip
>
> In nucleophilic addition reactions, electron-withdrawing groups increase the reactivity of carbonyl compounds, while electron-donating groups decrease their reactivity. This principle is crucial in determining the reactivity order of different compounds.

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9 Section - B

71. 37.8 g $N_2O_5$was taken in a 1 L reaction vessel and allowed to undergo the following reaction at 500 K:

$$2\mathrm{N_2O_5}(g) \rightarrow 2\mathrm{N_2O_4}(g) + \mathrm{O_2}(g)$$

The total pressure at equilibrium was found to be 18.65 bar. Then, K_p is:
Given:

$$R = 0.082\,\text{bar L mol}^{-1}\text{K}^{-1}$$

Solution: Step 1: evaluate the initial moles of N_2O_5.

Molar mass of $N_2O_5 = 2(14) + 5(16) = 28 + 80 = 108$ g/mol
Initial moles of N_2O_5, $n_0 = \frac{37.8}{108} = 0.35$ mol

Step 2: Set up the ICE table for partial pressures.

Let the initial pressure of N_2O_5 be P_0. Using the ideal gas law, $P_0V = n_0RT$, so
$P_0 = \frac{n_0RT}{V} = \frac{0.35\times0.082\times500}{1} = 14.35$ bar

Total pressure at equilibrium, $P_T = (P_0 - 2x) + 2x + x = P_0 + x$
Its given $P_T = 18.65$ bar, so $18.65 = 14.35 + x$, which means $x = 4.3$ bar

Step 3: evaluate the equilibrium partial pressures.
$P_{N_2O_5} = P_0 - 2x = 14.35 - 2(4.3) = 14.35 - 8.6 = 5.75$ bar $P_{N_2O_4} = 2x = 2(4.3) = 8.6$ bar
$P_{O_2} = x = 4.3$ bar

Step 4: evaluate Kp.

$$K_p = \frac{P_{N_2O_4}^2 \cdot P_{O_2}}{P_{N_2O_5}^2} = \frac{(8.6)^2 \cdot (4.3)}{(5.75)^2} = \frac{73.96 \cdot 4.3}{33.0625} = \frac{318.028}{33.0625} \approx 9.619$$

Then $K_p = 9.619$ Kp = ______ $\times10^{-2}$ 961.9×10^{-2} approximately 962×10^{-2} (nearest integer)
Therefore the closest integer is 962

> Quick Tip
>
> For the equilibrium constant in terms of pressure, use the partial pressures of the gases involved in the reaction, and apply the ideal gas law to evaluate the total moles at equilibrium.

---

72. Standard entropies of X_2, Y_2 and XY_5 are $70, 50,$ and 110 J K^{-1} mol^{-1} respectively. The temperature in Kelvin at which the reaction

$$\frac{1}{2}X_2 + \frac{5}{2}Y_2 \rightarrow XY_5 \quad \Delta H = -35\,\text{kJ mol}^{-1}$$

will be at equilibrium is (nearest integer):
Solution:
Step 1: evaluate the standard entropy change (ΔS) for the reaction.
The standard entropy change is evaluated using the formula:

$$\Delta S = S_{XY_5} - \left(\frac{1}{2}S_{X_2} + \frac{5}{2}S_{Y_2}\right)$$

Substitute the given entropy values:

$$\Delta S = 110 - \left(\frac{1}{2} \times 70 + \frac{5}{2} \times 50\right)$$

$$\Delta S = 110 - (35 + 125) = 110 - 160 = -50\,\text{J K}^{-1}\text{mol}^{-1}$$

Step 2: Use the equation for temperature at equilibrium.
At equilibrium, the change in Gibbs free energy (ΔG) is zero, and we use the following relation:

$$\Delta G = \Delta H - T\Delta S$$

Since $\Delta G = 0$ at equilibrium:

$$0 = \Delta H - T\Delta S$$

Rearranging to solve for T:

$$T = \frac{\Delta H}{\Delta S}$$

Substitute the known values for ΔH (in J/mol) and ΔS:

$$\Delta H = -35\,\text{kJ/mol} = -35000\,\text{J/mol}$$

$$T = \frac{-35000\,\text{J/mol}}{-50\,\text{J K}^{-1}\text{mol}^{-1}} = 700\,\text{K}$$

Therefore, the temperature at equilibrium is $\boxed{700}$ K.

Quick Tip

For equilibrium calculations, use the relation $\Delta G = \Delta H - T\Delta S$. At equilibrium, $\Delta G = 0$, so solving for T gives the temperature at equilibrium.

---

73. X g of benzoic acid on reaction with aqueous $NaHCO_3$ release CO_2 that occupied 11.2 L volume at STP. X is ____ g.

Correct Answer: (61) g

Solution: Step 1: Write the balanced chemical equation.

Benzoic acid (C_6H_5COOH) reacts with $NaHCO_3$ as follows:
$C_6H_5COOH(aq) + NaHCO_3(aq) \rightarrow C_6H_5COONa(aq) + H_2O(l) + CO_2(g)$

Step 2: evaluate the moles of CO_2 released.

At STP, 1 mole of any gas occupies 22.4 L. Therefore, the number of moles of CO_2 released is:

$n_{CO_2} = \frac{11.2\text{ L}}{22.4\text{ L/mol}} = 0.5$ mol

Step 3: evaluate the moles of benzoic acid.

From the balanced equation, 1 mole of benzoic acid reacts to produce 1 mole of CO_2. Therefore, the number of moles of benzoic acid is equal to the number of moles of CO_2:

$n_{\text{Benzoic Acid}} = n_{CO_2} = 0.5$ mol

Step 4: evaluate the mass of benzoic acid.

The molar mass of benzoic acid (C_6H_5COOH) is:

$6(12) + 5(1) + 12 + 2(16) + 1 = 72 + 5 + 12 + 32 + 1 = 122$ g/mol
The mass of benzoic acid is:

$X = n_{\text{Benzoic Acid}} \times \text{Molar Mass} = 0.5 \text{ mol} \times 122 \text{ g/mol} = 61 \text{ g}$

Therefore, X is 61 g.

Quick Tip

In such reactions, always use the stoichiometric relationship from the balanced chemical equation to convert volumes of gases into moles. Then use the molar mass to find the corresponding mass.

---

74. Among the following cations, the number of cations which will give characteristic precipitate in their identification tests with $K_4[\text{Fe(CN)}_6]$ is :

$$\text{Cu}^{2+}, \text{Fe}^{3+}, \text{Ba}^{2+}, \text{Ca}^{2+}, \text{NH}_4^+, \text{Mg}^{2+}, \text{Zn}^{2+}$$

Solution:
The characteristic precipitates formed with $K_4[\text{Fe(CN)}_6]$ are based on the ability of certain cations to form a precipitate with the ferrocyanide ion. The relevant reactions are:

Step 1: Identify the cations that give precipitates with $\mathbf{K_4[Fe(CN)_6]}$.

1.Cu^{2+}: Forms a blue precipitate of $Cu_2[Fe(CN)_6]$.
2. Fe^{3+}: Forms a blue precipitate of $\text{Fe}_4[\text{Fe(CN)}_6]_3$.
3. Zn^{2+}: Forms a white precipitate of $\text{Zn}_2[\text{Fe(CN)}_6]$.

Step 2: Identify the cations that do not give precipitates.
The following cations do not form characteristic precipitates with $K_4[\text{Fe(CN)}_6]$:
- Ba^{2+}, Ca^{2+}, NH_4, and Mg^{2+} do not react to form precipitates with ferrocyanide.
Conclusion:
The cations that form characteristic precipitates with $K_4[\text{Fe(CN)}_6]$ are $\text{Cu}^{\mathbf{2}}$, $\text{Fe}^{\mathbf{3}}$, and $\text{Zn}^{\mathbf{2}}$, which gives us a total of 3 cations.
Therefore, the correct answer is $\boxed{(3)}$.

Quick Tip

In tests with $K_4[Fe(CN)_6]$, only certain cations like Cu^2, Fe^3, and Zn^2 give characteristic precipitates. Cations such as Ba^2, Ca^2, NH, and Mg^2 typically do not react in this way.

---

75. Consider the following reaction occurring in the blast furnace.

$$Fe_3O_4(s) + 4CO(g) \rightarrow 3Fe(l) + 4CO_2(g)$$

'x' kg of iron is produced when 2.32×10^3 kg Fe_3O_4 and 2.8×10^2 **kg CO are brought together in the furnace.** The value of 'x' is ____ (nearest integer).

Correct Answer: (420) g

Solution:
Step 1: Convert masses to moles.
Moles of $Fe_3O_4 = \frac{2.32 \times 10^6 \text{ g}}{232 \text{ g/mol}} = 10^4$ mol Moles of CO $= \frac{2.8 \times 10^5 \text{ g}}{28 \text{ g/mol}} = 10^4$ mol

Step 2: Identify the limiting reactant.
From the balanced equation, 1 mole of Fe_3O_4 reacts with 4 moles of CO. The mole ratio of Fe_3O_4 to CO is 1:4.
The available mole ratio is $\frac{10^4}{10^4} = 1$. Since the reaction requires a ratio of 1:4, Fe_3O_4 is in excess and CO is the limiting reactant.

Step 3: evaluate the moles of Fe produced.
From the balanced equation, 4 moles of CO produce 3 moles of Fe. So, Moles of Fe $= \frac{3}{4} \times$ Moles of CO $= \frac{3}{4} \times 10^4 = 7.5 \times 10^3$ mol

Step 4: Convert moles of Fe to kg.
Mass of Fe = Moles of Fe $\times$ Molar mass of Fe Mass of Fe $= 7.5 \times 10^3$ mol $\times$ 56 g/mol $= 420 \times 10^3$ g $= 420$ kg
Therefore, the value of x is 420.

Quick Tip

In stoichiometry problems involving mass and moles, always make sure to use the correct molar masses and balance the equation to understand the relationships between reactants and products. You can evaluate the mass produced using the stoichiometric ratios.
